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Free choice, simplification, and Innocent Inclusion

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Abstract
We propose a modification of the exhaustivity operator from Fox (in: Sauerland and Stateva (eds) Presupposition and implicature in compositional semantics, Palgrave Macmillan, London, pp 71–120, 2007. https://doi.org/10.1057/9780230210752_4) that on top of negating all the Innocently Excludable alternatives affirms all the 'Innocently Includable' ones. The main result of supplementing the notion of Innocent Exclusion with that of Innocent Inclusion is that it allows the exhaustivity operator to identify cells in the partition induced by the set of alternatives (assign a truth value to every alternative) whenever possible. We argue for this property of 'cell identification' based on the simplification of disjunctive antecedents and the effects on free choice that arise as the result of the introduction of universal quantifiers. We further argue for our proposal based on the interaction of *only* with free choice disjunction.

Keywords Implicature · Exhaustification · Free choice · Simplification of disjunctive antecedents · Innocent Inclusion · Innocent Exclusion

1 Introduction

As is well known, a sentence like (1), where an existential modal takes scope above *or*, gives rise to the *free choice* (FC) inferences (1a) and (1b) (von Wright 1968; Kamp 1974). It is also well known that these inferences don't follow from standard

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assumptions about the semantics of *allowed* and *or*: $\Diamond(a \vee b)$ is equivalent to $\Diamond a \vee \Diamond b$ rather than the conjunction $\Diamond a \wedge \Diamond b$.[1]

(1) *Free choice disjunction*:
Mary is allowed to eat ice cream or cake. $\Diamond(a \vee b) \not\Leftrightarrow (\Diamond a \vee \Diamond b)$
a. $\rightsquigarrow$ Mary is allowed to eat ice cream. $\Diamond a$
b. $\rightsquigarrow$ Mary is allowed to eat cake. $\Diamond b$

Implicature-based accounts of FC (Kratzer and Shimoyama 2002; Fox 2007; Klinedinst 2007; Chemla 2009a; Franke 2011) maintain that the basic meaning of (1) can be stated as $\Diamond(a \vee b)$ using standard modal logic and that the FC inferences are explained by enriching the meaning with mechanisms that are independently motivated in the account of scalar implicatures.[2]

Implicature accounts of FC are motivated both on conceptual grounds—they do not involve altering the meaning of logical words—and on empirical grounds—they correctly predict the behavior of FC disjunction under negation (Alonso-Ovalle 2005). However, there are some lingering problems for such accounts, which will be the focus of this paper. First, when FC disjunction is embedded under *only*, the FC inferences become part of *only*'s presupposition (Alxatib 2014). As (2) shows, the inferences project out of a question as though they were presuppositions. At the same time, deriving FC inferences in the scope of *only* (as an embedded implicature) leads to further problems; specifically, it yields an assertive component which is weaker than attested.

(2) *FC disjunction under* only:
Are we only allowed to eat [ice cream or cake]$_F$?
a. $\rightsquigarrow$ We are allowed to eat ice cream.
b. $\rightsquigarrow$ We are allowed to eat cake.

Second, implicature approaches do not predict the distribution of *universal free choice* inferences such as those in (3a) and (3b). Specifically, under current implicature-based accounts such inferences must be derived by embedded exhaustification, but there is evidence that a global derivation must be available as well (Chemla 2009b).

(3) *Universal FC*:
Every boy is allowed to eat ice cream or cake. $\forall x \Diamond(Px \vee Qx)$
a. $\rightsquigarrow$ Every boy is allowed to eat ice cream. $\forall x \Diamond Px$
b. $\rightsquigarrow$ Every boy is allowed to eat cake. $\forall x \Diamond Qx$

[1] Here and throughout the paper we conflate logical forms and their truth conditions; e.g., $\Diamond(a \vee b)$ will at times stand for a logical form in which an existential modal takes scope over disjunction and at other times for the corresponding truth conditions. We distinguish between the two only if we think things might be confusing otherwise.

[2] We refer to accounts of FC as "implicature accounts" when they propose the same mechanism for FC and for run-of-the-mill scalar implicatures. This terminological choice (which we adopt from much current literature) might be confusing at times, as some of the accounts, like our own, do not take scalar implicatures to be implicatures in the traditional Gricean sense (but rather logical entailments of a potentially ambiguous sentence under one of its parses).

Third, a similar case of FC inferences which is not predicted by implicature accounts has been discussed by Nouwen (2017);[3] this is a case where a universal quantifier intervenes between the existential modal and disjunction:

(4) The teacher is OK with every student either talking to Mary or to Sue. $\Diamond\forall x(Px \vee Qx)$

a. $\rightsquigarrow$ The teacher is OK with every student talking to Mary. $\Diamond\forall x Px$

b. $\rightsquigarrow$ The teacher is OK with every student talking to Sue. $\Diamond\forall x Qx$

Fourth, the phenomenon of *simplification of disjunctive antecedents* (SDA), exemplified in (5), has been argued to pattern like FC, motivating a unified explanation. But such a unified explanation is not provided under all implicature accounts of FC (though see Klinedinst 2007; Franke 2011, which we will touch on in Sect. 7).

(5) *Simplification of disjunctive antecedents (SDA)*:
If you eat ice cream or cake, you will feel guilty. $(p \vee q) \,\Box\!\!\rightarrow r$

a. $\rightsquigarrow$ If you eat ice cream, you will feel guilty. $p \,\Box\!\!\rightarrow r$

b. $\rightsquigarrow$ If you eat cake, you will feel guilty. $q \,\Box\!\!\rightarrow r$

Our contribution in this paper is to propose a modification of Fox (2007), and to argue that it can provide the basis for a solution to these problems. Fox has defined an exhaustivity operator, $\mathcal{E}xh$, based on the notion of Innocent Exclusion. The results of applying that operator to a proposition and a set of alternatives are almost equivalent to the results of the algorithm proposed in Sauerland (2004) as a characterization of the outcome of neo-Gricean reasoning. One argument in favor of implementing Innocent Exclusion within a grammatical theory, namely letting $\mathcal{E}xh$ have a syntactic life, was that, unlike the neo-Gricean implementation, the grammatical one predicted recursive application of $\mathcal{E}xh$ to be available, which derived FC inferences.

The perspective put forth in the current paper involves a more radical departure from neo-Gricean reasoning.[4] Instead of merely negating alternatives, we propose that $\mathcal{E}xh$ "attempts", whenever possible, to assign a truth value to every alternative of its prejacent (i.e., to match alternatives with cells in the partition induced by the set of alternatives).[5] Our view of $\mathcal{E}xh$ as identifying cells leads to its definition

[3] The problem Nouwen (2017) focuses on is FC with ability modals. As we will see, this problem will not be resolved under our approach without stipulations about the nature of the alternatives. See Sect. 6.

[4] Note that under an $\mathcal{E}xh$-based theory of scalar implicatures, there is no reason to expect the nature of the operator to mimic neo-Gricean reasoning. In fact, if it did, this would be a suspicious coincidence which would cry for an explanation. Of course, it would be preferable for the properties of the operator to have some conceptual grounding. In Fox (2007, 2014, 2016, 2018b), it is suggested that the operator's properties should follow from the function it plays, namely allowing speakers to efficiently convey all of their beliefs about a topic of conversation (characterized by a partition). Cell identification meets this design specification in many cases. Specifically, it allows opinionated speakers to convey all of their relevant beliefs in response to a question Q while meeting an independent formal constraint on answers (Fox 2018a, b). See also the discussion around (23) below.

[5] Implicature calculation as resulting in cell identification has been pursued in Franke's (2011) game-theoretic approach, which is essentially a cell identification view of scalar implicatures. However, without various complications, Franke fails to derive several basic facts, such as FC disjunction with more than two disjuncts, universal FC inferences, and SDA with more than two disjuncts. Furthermore, he crucially relies on the assumption of equal priors for deriving FC (see Fox and Katzir 2018 for discussion).

as an operator which assigns *false* to certain alternatives—what Fox calls Innocent Exclusion—and furthermore assigns *true* to some other alternatives—what we will call Innocent Inclusion. We will argue that the key to analyzing examples (3)–(5) lies in letting $\mathcal{E}xh$ identify cells whenever possible. As for (2), the distinction between Innocent Exclusion and Inclusion will allow us to claim that *only* too, just like $\mathcal{E}xh$, involves Inclusion; the only difference between the two operators would be that *only* presupposes the 'Innocently Includable' alternatives, while $\mathcal{E}xh$ asserts them.

The structure of the paper is as follows: In Sect. 2 we discuss motivations for the implicature account of FC. We then present our proposal in Sect. 3, which, as just mentioned, builds on Fox's notion of Innocent Exclusion and introduces in addition the notion of Innocent Inclusion; we show that together they form a mechanism that identifies cells whenever possible, thus providing a direct derivation of FC disjunction. The core argument of the paper is then discussed in Sects. 4–8, where the four issues mentioned above, exemplified in (2)–(5), are argued to provide empirical support for Innocent Inclusion. Finally, Sect. 9 identifies some remaining issues that we have to leave for future research.

2 The implicature account of FC

2.1 Motivation for an implicature account

Alonso-Ovalle (2005), following Kratzer and Shimoyama (2002), argues that the FC inference from (1) to (1a,b) should be treated as a scalar implicature, due to the consequences of embedding the basic construction under negation, as in (6).

(6) John isn't allowed to eat ice cream or cake. $\neg\Diamond(a \vee b)$

a. $\not\approx$ It's not the case that John is both allowed to eat ice cream *and* allowed to eat cake (but maybe he's allowed one of them). $\neg(\Diamond a \wedge \Diamond b)$

b. $\approx$ It's not the case that John is allowed to eat ice cream and it's not the case that he is allowed to eat cake. $\neg\Diamond a \wedge \neg\Diamond b$

A standard translation of the relevant sentences into modal logic will provide the correct interpretation when embedding under negation is involved: $\neg\Diamond(a \vee b) \Leftrightarrow \neg\Diamond a \wedge \neg\Diamond b$. If the conjunctive interpretation of the unembedded constructions could be derived by the theory that derives scalar implicatures, it would have both the conceptual merit of keeping the underlying logic intact and the empirical merit of predicting the correct result for embedding under negation (as scalar implicatures are rarely computed under negation).

2.2 A novel argument from VP-ellipsis

To strengthen the argument, we point out here that FC behaves similarly to scalar implicatures under VP-ellipsis. In VP-ellipsis constructions like (7), where the elided material is under negation but the antecedent isn't, the salient interpretation is one where the antecedent is interpreted conjunctively, having an FC meaning, while the elided material is interpreted disjunctively, lacking an FC meaning.

(7) Mary is allowed to eat ice cream or cake, and John isn't ~~allowed to eat ice cream or cake~~. ≈

a. Mary is allowed to eat ice cream *and* allowed to eat cake, and
b. John isn't allowed to eat ice cream and he isn't allowed to eat cake.

Such behavior under ellipsis is typical for scalar implicatures. For example, in (8) the antecedent VP contains *some*, which is intuitively interpreted exhaustively as *some but not all*, while in the elided VP *some* is interpreted non-exhaustively under negation (see Fox 2004 for a similar data point, attributed to Tamina Stephenson, p.c.).

(8) Mary solved some of the problems, and John didn't ~~solve some of the problems~~. ≈

a. Mary solved *some but not all* of the problems, and
b. John didn't solve *any* of the problems.

Within the grammatical approach to scalar implicatures (Chierchia et al. 2012), such facts are explained even under the strictest theories of ellipsis, which demand semantic identity between an elided constituent and a pronounced antecedent (Sag 1976; Williams 1977), since the exhaustivity operator $\mathcal{E}xh$ that generates the *some but not all* or the FC inferences can be base generated above the antecedent.[6]

Example (7) is especially problematic for an ambiguity approach to FC such as Aloni (2007); within such an approach the semantics of *allowed to eat ice cream or cake* is different when under negation than when unembedded. Hence it is impossible to derive the meaning in (7a,b) while at the same time respecting the semantic identity required in VP-ellipsis between the antecedent and the elided material.[7]

2.3 Comments on other approaches to FC

As William Starr has pointed out to us, the argument from ellipsis doesn't carry over (at least not straightforwardly) to other accounts with a non-standard semantics which,

[6] And, of course, they are also explained when weaker demands are made, as in Rooth (1992) and Heim (1996)—the 'parallelism domain' for ellipsis (the domain of $\sim$ in Rooth 1992, Heim 1996) need not contain $\mathcal{E}xh$. If this operator were forced to be inside the parallelism domain, we would expect the implicature to be derived for the elided material as well; see Crnič (2015) for arguments that the presence of $\mathcal{E}xh$ inside the parallelism domain indeed has the predicted consequences. We will return to the issue of FC with ellipsis and the relevance of exhaustivity to parallelism in Sects. 5.3 and 9.2.

[7] A similar argument against ambiguity comes from non-monotonic contexts (see Bar-Lev 2018, Chap. 1):

(i) Exactly two girls are allowed to eat ice cream or cake.

a. $\rightsquigarrow$ Two girls are both allowed to eat ice cream and allowed to eat cake.
b. $\rightsquigarrow$ No more than two girls are allowed to eat ice cream or cake.

One might consider the following fix for salvaging the ambiguity approach, in light of (7) and (i): *allowed A or B* gives rise to a special kind of ambiguity, which requires truth on all of its resolutions (along the lines of the ambiguity approach to homogeneity phenomena in Spector 2013; Križ and Spector 2017; Spector 2018). Since it does not seem to extend to every kind of ambiguity resolution we do not pursue this route.

unlike Aloni (2007), don't rely on ambiguity (see recently Starr 2016; Aloni 2016; Willer 2018). In order to deliver the right truth conditions for sentences like (6) without assuming ambiguity, those approaches rely on defining truth and falsity conditions separately (bilaterally) for *allowed* and for *or*.[8] Once this is done, negation can be defined so that it interacts in the desired way with the bilateral meaning of *allowed to eat ice cream or cake*. As Aloni (2016) points out, this considerably weakens the explanatory power of those approaches, since the "behaviour under negation is postulated rather than predicted: allowing to pre-encode what should happen under negation, bilateral systems are more descriptive than explanatory."[9,10]

Our goal in this paper, then, is to seek an explanatory account of FC. As mentioned in the Introduction, we believe that deriving FC via an exhaustivity operator can in principle meet this goal. However, there are remaining empirical challenges that motivate modification of the theory of $\mathcal{E}xh$.

Some challenges to a theory of FC as an implicature are not fully addressed in this paper. First, it has been claimed that FC inferences behave differently than other scalar implicatures (e.g., Barker 2010; Chemla and Bott 2014). Second, it has been claimed that FC inferences are attested even when disjunction takes scope above the modal (i.e., with sentences of the form $\Diamond a \vee \Diamond b$), which isn't predicted by implicature accounts. We briefly discuss the first issue in Sect. 9.1; both are discussed more fully in Bar-Lev (2018, Chap. 2).

3 Proposal: Innocent Inclusion

3.1 Disjunction and its alternatives

A key point in analyzing FC disjunction is understanding the distinction between (9) and (10). While from simple disjunction we infer the exclusive inference in (9a), for FC disjunction we infer what we might call the opposite inference in (10a), namely FC.

(9) *Simple disjunction*:
Mary ate ice cream or cake. $a \vee b$
a. $\rightsquigarrow$ Mary didn't eat both ice cream and cake. $\neg(a \wedge b)$

(10) *FC disjunction*:
Mary is allowed to eat ice cream or cake. [=(1)] $\Diamond(a \vee b)$
a. $\rightsquigarrow$ Mary is allowed to eat ice cream and allowed to eat cake. $\Diamond a \wedge \Diamond b$

To see if we can view both the exclusive inference in (9a) and the FC inference in (10a) as scalar implicatures, we should start by asking what alternatives are generated in

[8] This is not strictly speaking true of Starr (2016), whose proposal nonetheless involves non-standard modifications of logical operators, a move we are trying to avoid.

[9] In addition, some of those approaches (Starr 2016; Aloni 2016) predict FC inferences when disjunction takes wide scope above the modal, a result argued against in Bar-Lev (2018, Chap. 2).

[10] This criticism is reminiscent of Schlenker's (2009) criticism of dynamic semantics approaches.

each case (since scalar implicatures can only be determined when alternatives are specified).

Various arguments have been provided for the following analysis: disjunction gives rise to disjunctive alternatives—that is, alternatives where the disjunction is replaced by the individual disjuncts (see, e.g., Kratzer and Shimoyama 2002; Sauerland 2004; Katzir 2007)[11]—and to a conjunctive alternative—an alternative where disjunction is replaced with conjunction. When this analysis is applied to the sentences in (9) and (10), we generate the alternatives in (11a) and (11b), respectively.[12]

(11) a. *Set of alternatives for simple disjunction*:

$$Alt(a \vee b) = \{ \underbrace{a \vee b}_{\text{Prejacent}}, \underbrace{a, b}_{\text{Disjunctive alts.}}, \underbrace{a \wedge b}_{\text{Conjunctive alt.}} \}$$

b. *Set of alternatives for FC disjunction*:

$$Alt(\Diamond(a \vee b)) = \{ \underbrace{\Diamond(a \vee b)}_{\text{Prejacent}}, \underbrace{\Diamond a, \Diamond b}_{\text{Disjunctive alts.}}, \underbrace{\Diamond(a \wedge b)}_{\text{Conjunctive alt.}} \}$$

Looking at these sets of alternatives in light of the inferences we get in (9) and (10), we can see a striking difference between simple disjunction and FC disjunction with regard to the conjunction of their disjunctive alternatives:

(12) *Observation*:

a. From simple disjunction we infer that the conjunction of the disjunctive alternatives $(a \wedge b)$ is *false*.

b. From FC disjunction we infer that the conjunction of the disjunctive alternatives $(\Diamond a \wedge \Diamond b)$ is *true*.

What distinguishes the two cases and yields the opposite results in (12)? We will adopt the answer in (13) provided by Fox (2007), Chemla (2009a), and Franke (2011), according to which *closure under conjunction* is the distinguishing property: whereas $Alt(a \vee b)$ is closed under conjunction, $Alt(\Diamond(a \vee b))$ is not.

[11] There are several reasons for assuming that disjunctive alternatives are generated: First, their existence provides an explanation for the fact that from a sentence like (i) we infer (ia) and (ib), i.e., the negation of the disjunctive alternatives (*You are required to solve problem A*, *You are required to solve problem B*). See Sauerland (2004), Spector (2007), Katzir (2007), Fox (2007); as well as Sect. 5.5.

(i) You are required to solve problem A or problem B.

a. $\rightsquigarrow$ You are not required to solve problem A.

b. $\rightsquigarrow$ You are not required to solve problem B.

Second, if FC inferences are to be treated as scalar implicatures, as we assume here, disjunctive alternatives are required. Third, the independently motivated structural approach to the generation of alternatives (Katzir 2007) predicts such alternatives to be generated, being the result of replacing a constituent (*P or Q*) with a sub-constituent (*P*, *Q*).

[12] To avoid clutter, we ignore here and throughout the paper alternatives generated by replacing $\Diamond$ with $\Box$ which do not interfere with the derivation of FC. We discuss whether such alternatives should be derived to begin with in Sect. 5.5.

(13) a. The conjunction of the disjunctive alternatives a and b, i.e., $a \wedge b$, *is* a member of $Alt(a \vee b)$.
b. The conjunction of the disjunctive alternatives $\Diamond a$ and $\Diamond b$, i.e., $\Diamond a \wedge \Diamond b$, *is not* a member of $Alt(\Diamond(a \vee b))$.

To see how this distinction might yield opposite results for the two cases, let us focus on the account of FC in Fox (2007).

3.2 Innocent Exclusion

Within the version of the grammatical theory assumed by Fox (2007), scalar implicatures are generated by applying a covert exhaustivity operator, $\mathcal{E}xh$, akin to overt *only*. This operator takes a prejacent and a set of alternatives. What should it return as output? If we let $\mathcal{E}xh$ assign *false* to every alternative, we would sometimes get a contradictory result, even if we are "smart enough" and restrict the procedure to those alternatives whose falsity is consistent with the prejacent. To see this, consider the case of exhaustifying $a \vee b$ with respect to $Alt(a \vee b)$. Since both a and b are non-weaker than $a \vee b$, $\mathcal{E}xh$ would assign them *false* and yield the contradiction $(a \vee b) \wedge \neg a \wedge \neg b$.

$\mathcal{E}xh$ must be a bit "smarter" than this: it must find a way to exclude as many alternatives as possible, while caring about overall consistency. To achieve this, Fox (2007) introduces the notion of Innocent Exclusion.

(14) *Innocent Exclusion procedure*:
a. Take all maximal sets of alternatives that can be assigned *false* consistently with the prejacent.
b. Only exclude (i.e., assign *false* to) those alternatives that are members in all such sets—the *Innocently Excludable* (=IE) alternatives.

Let us show now how Innocent Exclusion avoids contradiction when applied to $a \vee b$ and to $\Diamond(a \vee b)$. To apply Innocent Exclusion to $a \vee b$ we first have to identify the maximal sets of alternatives in $Alt(a \vee b)$ that can be assigned *false* consistently with the prejacent. There are two such sets: $\{a, a \wedge b\}$ and $\{b, a \wedge b\}$. The second step, by the Innocent Exclusion procedure, is to exclude the alternatives which are in all of those sets; there is only one such alternative, $a \wedge b$, which is thus the only IE alternative.

A parallel result is derived by applying Innocent Exclusion to $\Diamond(a \vee b)$. The maximal sets of alternatives in $Alt(\Diamond(a \vee b))$ that can be assigned *false* consistently with the prejacent are $\{\Diamond a, \Diamond(a \wedge b)\}$ and $\{\Diamond b, \Diamond(a \wedge b)\}$, and consequently the only IE alternative is $\Diamond(a \wedge b)$. The result of Innocent Exclusion for the two cases is represented schematically in Fig. 1. At this stage, for simple disjunction we derive that the prejacent $a \vee b$ is true and that the conjunctive alternative $a \wedge b$ is false, and similarly for FC disjunction: the prejacent $\Diamond(a \vee b)$ is true and the conjunctive alternative $\Diamond(a \wedge b)$ is false.[13] This gives us the first hint as to why we might derive opposite results for the two cases.

[13] As pointed out by Simons (2005), FC inferences are not always accompanied by the inference that excludes the conjunctive alternative $\Diamond(a \wedge b)$. Our derivation of FC in what follows does not depend on deriving the falsity of the conjunctive alternative (in fact it follows even if this alternative is omitted altogether from the set of alternatives). See Sect. 9.1 and Bar-Lev (2018, Chap. 2) for reasons why this alternative will be assigned *false* only if it's taken to be relevant. Of course, one would then need to say why the same reasoning cannot apply to simple disjunction: why can't the conjunctive alternative in this

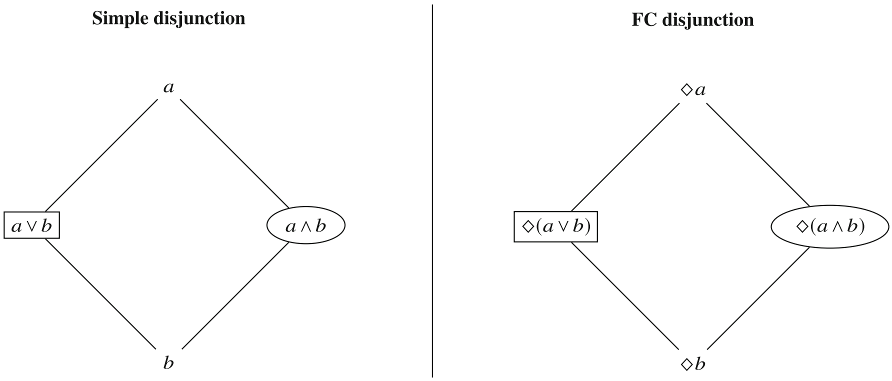


Fig. 1 Results of Innocent Exclusion for simple and FC disjunction. The lines represent entailment relations from right to left; the prejacent is marked with ▭ and the IE alternatives with ◯.

Since $Alt(a \vee b)$ is closed under conjunction, the output of applying Innocent Exclusion to $a \vee b$—which ensures the falsity of $a \wedge b$—*is not compatible* with the truth of both disjunctive alternatives a and b. In contrast, since $Alt(\Diamond(a \vee b))$ is not closed under conjunction, the output of applying Innocent Exclusion to $\Diamond(a \vee b)$—which ensures the falsity of $\Diamond(a \wedge b)$ (but crucially not the falsity of $\Diamond a \wedge \Diamond b$)—*is compatible* with both disjunctive alternatives $\Diamond a$ and $\Diamond b$ being true.[14]

But FC is not yet derived. Given what we have said so far we can only explain why it would be possible in principle to derive a conjunctive meaning for disjunction when embedded under an existential modal, but not for simple disjunction: a conjunctive meaning is consistent with the result of applying Innocent Exclusion in the case of FC disjunction but not in the case of simple disjunction. We still have to find a way to actually derive the conjunctive meaning.

Footnote 13 continued
case be irrelevant, and thus lead to a conjunctive reading for simple disjunction? The answer that has been given in the literature (e.g., Fox and Katzir 2011) is based on the assumption that relevance is closed under Boolean operations (conjunction and negation). It is thus possible for $\Diamond a$ and $\Diamond b$ to be relevant without $\Diamond(a \wedge b)$ being relevant; in contrast, once a and b are relevant, $a \wedge b$ must be relevant as well. A simplifying assumption in the discussion here and in what follows is that all formal alternatives are relevant, but this should not be seen as an empirical claim that we always infer the falsity of the IE alternatives. See Fox and Katzir (2011) and Singh et al. (2016) for relevant discussion.

[14] Lack of closure under conjunction is a necessary, though not sufficient, requirement for compatibility with a conjunctive inference. There are cases that meet the requirement in which a conjunctive inference is nevertheless incompatible with the negation of all IE alternatives. To see this, consider Fox's (2007) explanation of the fact that singular indefinites (in contrast to existential modals and plural indefinites) do not license conjunctive inferences: *Some boy ate ice cream or cake* does not lead to the inference that some boy ate ice cream and some boy ate cake. The explanation is based on the observation that singular indefinites lead to a potential *only one* scalar implicature. (*Someone in the class did the homework* can lead to the inference that only one person did the homework.) This potential scalar implicature demands populating the space of alternatives with alternatives whose negation amounts to the proposition that there aren't two boys who ate ice cream or cake. This proposition, together with the negation of the conjunctive alternative (i.e., the proposition that no boy ate ice cream and cake) is incompatible with the conjunctive inference.

The Innocent Exclusion procedure in (14) leads to the lexical entry for the exhaustivity operator $\mathcal{E}xh^{\text{IE}}$ in (15a): given a set of alternatives C and a prejacent p, it would assign *true* to the prejacent and *false* to all the IE alternatives (defined in (15b)).

(15) *Innocent Exclusion–based exhaustivity operator*: (Fox 2007)

a. $[\![\mathcal{E}xh^{\text{IE}}]\!](C)(p)(w) \Leftrightarrow p(w) \wedge \forall q \in IE(p, C)[\neg q(w)]$

b. $IE(p, C) = \bigcap\{C' \subseteq C : C'$ is a maximal subset of C, s.t. $\{\neg q : q \in C'\} \cup \{p\}$ is consistent$\}$

In the next section we introduce the notion of *Innocent Inclusion*, and suggest a different lexical entry for $\mathcal{E}xh$ than (15a), one that implements both Innocent Exclusion and Innocent Inclusion and can directly derive FC inferences.

3.3 Introducing Innocent Inclusion

How do we derive the FC inferences for FC disjunction? This is where we depart from Fox (2007). The FC inferences $\Diamond a$ and $\Diamond b$ are derived by Fox *indirectly*, by applying $\mathcal{E}xh^{\text{IE}}$ recursively. Our proposal is to define $\mathcal{E}xh$ differently from (15a)—to strengthen it in a way that would yield the FC inferences *directly*. Before presenting our proposal, we first illustrate in Sect. 3.3.1 how Fox (2007) derives FC with a recursive application of $\mathcal{E}xh^{\text{IE}}$. As we have mentioned in the Introduction, Fox's approach faces some empirical challenges, such as universal FC and SDA. We will argue in Sect. 3.3.2 that these challenges can be seen to reveal different instantiations of a single generalization that Fox's account fails to capture. We then move on in Sect. 3.3.3 to demonstrating that the generalization is predicted when the notion of Innocent Inclusion is incorporated into the definition of $\mathcal{E}xh$.

3.3.1 Deriving FC with recursive $\mathcal{E}xh^{\text{IE}}$

Applying $\mathcal{E}xh^{\text{IE}}$ once to $\Diamond(a \vee b)$ we get (16), as shown in Fig. 1.

(16) $\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b) \Leftrightarrow \Diamond(a \vee b) \wedge \neg\Diamond(a \wedge b)$. (where $C = Alt(\Diamond(a \vee b))$)

We can now apply $\mathcal{E}xh^{\text{IE}}$ once more, over $\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b)$. The alternatives for the second application of $\mathcal{E}xh^{\text{IE}}$ are the exhaustified alternatives of $\Diamond(a \vee b)$:

(17) $Alt(\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b)) =$
$$\{\underbrace{\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b)}_{\text{Prejacent}}, \underbrace{\mathcal{E}xh^{\text{IE}}_C \Diamond a\ ,\ \mathcal{E}xh^{\text{IE}}_C \Diamond b}_{\text{Disjunctive alts.}}, \underbrace{\mathcal{E}xh^{\text{IE}}_C \Diamond(a \wedge b)}_{\text{Conjunctive alt.}}\}$$

Computing the exhaustified alternatives, we get the following:

(18) $Alt(\mathcal{E}xh^{\text{IE}}_C \Diamond(a \vee b)) =$
$$\{\underbrace{\Diamond(a \vee b) \wedge \neg\Diamond(a \wedge b)}_{\text{Prejacent}}, \underbrace{\Diamond a \wedge \neg\Diamond b\ ,\ \Diamond b \wedge \neg\Diamond a}_{\text{Disjunctive alts.}}, \underbrace{\Diamond(a \wedge b)}_{\text{Conjunctive alt.}}\}$$

Now, applying $\mathcal{E}xh^{\text{IE}}$ with respect to this set we get (19) (since all the alternatives are IE):

(19) $\mathcal{E}xh^{\text{IE}}_{Alt(\mathcal{E}xh^{\text{IE}}_{C}\Diamond(a\vee b))}\mathcal{E}xh^{\text{IE}}_{C}\Diamond(a\vee b)$
$$\Leftrightarrow \underbrace{\Diamond(a\vee b)\wedge\neg\Diamond(a\wedge b)}_{\text{Prejacent}}\wedge\underbrace{\neg(\Diamond a\wedge\neg\Diamond b)\wedge\neg(\Diamond b\wedge\neg\Diamond a)}_{\text{Negation of disjunctive alternatives}}$$
$$\Leftrightarrow \underbrace{\Diamond(a\vee b)\wedge\neg\Diamond(a\wedge b)}_{\text{Prejacent}}\wedge\underbrace{\Diamond a\wedge\Diamond b}_{\text{FC inferences}}$$

3.3.2 Cell identification

Note that the result of the recursive application of $\mathcal{E}xh^{\text{IE}}$ in (19) entails that the prejacent $\Diamond(a\vee b)$ and the disjunctive alternatives $\Diamond a$ and $\Diamond b$ are true, and that the conjunctive IE alternative $\Diamond(a\wedge b)$ is false. That is, recursive exhaustification, in this particular case, determines the truth value of every alternative in C. But recursive exhaustification of $\mathcal{E}xh^{\text{IE}}$ does not always have this property. Our argument for a modification of $\mathcal{E}xh$ will be based on the observation that it doesn't in certain cases where it should. To understand the argument it is useful to define a function, $Cell(p, C)$, that takes a prejacent p and a set of alternatives C and returns a proposition that is true if all members of $IE(p, C)$ are false and all other members of C are true:[15]

(20) $Cell(p, C) = p \wedge \bigwedge\{\neg q : q \in IE(p, C)\} \wedge \bigwedge(C \setminus IE(p, C))$

This proposition, of course, determines a truth value for every member of C. Our empirical claim is that whenever this determination is consistent (whenever the output of the function is distinct from $\bot$), it should be the output of exhaustification.[16] Given a prejacent p and a set of alternatives C, the proposition returned by *Cell* is true if and only if the prejacent is true, all the IE alternatives are false, and all the non-IE alternatives are true. In the case of $\Diamond(a\vee b)$ recursive application of $\mathcal{E}xh^{\text{IE}}$ turns out equivalent to the application of *Cell*, that is, the following holds:

(21) For $p = \Diamond(a\vee b)$ and $C = Alt(\Diamond(a\vee b))$:
$\mathcal{E}xh^{\text{IE}^2}_{C}(p) \Leftrightarrow Cell(p, C)$
(where $\mathcal{E}xh^{\text{IE}^2}$ stands for two applications of $\mathcal{E}xh^{\text{IE}}$)

The case of $\Diamond(a\vee b)$, we will argue, is but one manifestation of the following generalization:

[15] To understand why we call this function *Cell*, consider the partition induced by the set of alternatives, that is, the partition of logical space into sets of worlds that assign the same truth value to every member of C—$Partition(C)$. $Partition(C)$ can be written in a format reminiscent of our definition of $Cell(p, C)$ as the set of non-contradictory propositions, each of which is the conjunction of a subset C' of C and the conjunction of the negations of all remaining alternatives (i.e., alternatives in $C \setminus C'$):

(i) $Partition(C) = \{p : p \neq \bot \wedge \exists C' \subseteq C[p = \bigwedge\{\neg q : q \in C'\} \wedge \bigwedge(C \setminus C')]\}$

It is easy to see that $Cell(p, C)$ is either a contradiction or a cell in $Partition(C)$.

[16] As long as $p \in C$ and C is finite, the first conjunct p on the right-hand side of (20) is redundant. See fn. 20.

(22) *Cell identification (when possible)*:
Let S be a sentence with denotation p and C be the set of denotations of its alternatives. If $Cell(p, C)$ is not a contradiction, then S can have $Cell(p, C)$ as a strengthened meaning.

We will discuss in Sects. 5–8 several cases demonstrating that (22) is not predicted to hold if our strengthening tool $\mathcal{E}xh$ is $\mathcal{E}xh^{\mathrm{IE}}$; we will, however, provide evidence that it holds empirically. This evidence will support the new definition of $\mathcal{E}xh$ that derives (22), to which we now turn.

3.3.3 Innocent Inclusion and cell identification

Assuming that (22) is indeed an empirical generalization and one that does not follow from recursive exhaustification, a shift in perspective is called for. We will suggest that exhaustification should not only lead to the exclusion of as many alternatives as possible (as in $\mathcal{E}xh^{\mathrm{IE}}$); it should also lead to the "inclusion" of as many alternatives as possible given what has been excluded.

More specifically, we suggest a direct implementation of this idea, one in which $\mathcal{E}xh$ has a dual role: it doesn't only *exclude* certain alternatives, assigning them the truth value *false*; it also *includes* some other alternatives, assigning them the truth value *true*.

At this point we would like to mention the underlying conception that has guided our thinking, namely (23).

(23) *Possible underlying conception*:
Exhaustifying p with respect to a set of alternatives C should get us as close as possible to a cell in the partition induced by C.

Namely, the goal of $\mathcal{E}xh$ is to come as close as possible to an assignment of a truth value to every alternative, i.e., to a cell in the partition that the set of alternatives induces (see footnotes 4 and 15). In other words, $\mathcal{E}xh$ is designed such that, when possible, it would yield a complete answer to the question formed by the set of alternatives.[17] If this conception is correct, one would think that $\mathcal{E}xh$ shouldn't only exclude, i.e., assign *false* to as many alternatives as possible, but should also include, i.e., assign *true* to as many alternatives as possible once the exclusion is complete.

What are the alternatives we want $\mathcal{E}xh$ to include? One possibility we might entertain is that $\mathcal{E}xh$ blindly includes all non-IE alternatives. But this would sometimes lead to contradictions: exhaustifying $a \vee b$ with respect to $Alt(a \vee b)$ would yield a contradiction, because including a and b, which are both non-IE, would contradict the derived falsity of $a \wedge b$. In other words, Inclusion just like Exclusion must apply *innocently*, so as to avoid contradictions. We thus suggest the procedure of Innocent Inclusion in (24).

[17] See Fox (2018b) for the relevance of this notion of exhaustivity as cell identification to issues having to do with the semantics of questions.

(24) *Innocent Inclusion procedure*:
  a. Take all maximal sets of alternatives that can be assigned *true* consistently with the prejacent and the falsity of all IE alternatives.
  b. Only include (i.e., assign *true* to) those alternatives that are members in all such sets—the *Innocently Includable* (=II) alternatives.

Note the similarity between Innocent Exclusion and Innocent Inclusion. Innocent Inclusion is only different from Innocent Exclusion in two respects: (i) that we include instead of exclude, and (ii) that we check for consistency not only with respect to the prejacent but also with respect to the falsity of all the IE alternatives.[18]

Let us see now how Innocent Inclusion applies to simple and FC disjunction to derive the desired results. Having Innocent Inclusion changes nothing for simple disjunction: the only II alternative is the prejacent $a \vee b$. This is so since the maximal sets of alternatives that are consistent with the truth of $a \vee b$ (the prejacent) and the falsity of $a \wedge b$ (the IE alternative) are $\{a \vee b, a\}$ and $\{a \vee b, b\}$, and the only member in their intersection is the prejacent $a \vee b$.

For FC disjunction, on the other hand, we derive the desired FC inferences with our procedure, since $\Diamond a$ and $\Diamond b$ are II. In this case, all the alternatives which are not IE are together consistent with the truth of $\Diamond(a \vee b)$ (the prejacent) and the falsity of $\Diamond(a \wedge b)$ (the IE alternative). That is, we only have one maximal set of alternatives to consider, $\{\Diamond(a \vee b), \Diamond a, \Diamond b\}$, and consequently all alternatives within this set are II. Therefore applying Innocent Exclusion and Innocent Inclusion yields a cell in the partition (a complete answer) in this case: the output tells us of every alternative whether it is true or false. The results of Innocent Exclusion and Innocent Inclusion for the two cases are represented schematically in Fig. 2.

We have seen that applying Innocent Exclusion and Innocent Inclusion yields the desired results for simple and FC disjunction: an exclusive *or* meaning for simple disjunction and an FC meaning for FC disjunction. For these two cases, the results we derive are identical to those Fox (2007) derives with the recursive application of $\mathcal{E}xh^{\text{IE}}$.

The lexical entry of the exhaustivity operator we are assuming here, $\mathcal{E}xh^{\text{IE+II}}$, implements both Innocent Exclusion and Innocent Inclusion. We first define the sets of IE and II alternatives: the set of IE alternatives in (25a) remains as in Fox (2007), and

[18] Why do we have to consider the set of IE alternatives for determining the set of II alternatives, and not vice versa? Let us consider what would happen if we first considered what's II: take for example the sentence *Some boy came* and its alternative *Every boy came*. If we were to include first, we would derive that the alternative *Every boy came* is true, namely exhaustifying over *Some boy came* would yield a meaning equivalent to *Every boy came*. This would make for a very inefficient tool to use in conversation: by choosing an utterance from the set of alternatives {*Some boy came*, *Every boy came*} an opinionated speaker would only be able to convey one epistemic state she might be in (one cell in the partition); she would not be able to convey an epistemic state that entails *Some but not all boys came*. In other words, this procedure would be a bad tool for answering questions (see Fox 2018b) since, no matter what utterance we choose from the set of alternatives, we get the same cell in the partition. (And this problem, of course, would generalize to every case in which the set of alternatives, C, is consistent. In every such case each member of C would be mapped by exhaustification to the same cell: $\bigwedge C$.) One can also prove that there are no cases with the reverse property, where point-wise exhaustification of the members of C would yield more distinctions if Inclusion is done before Exclusion. We can, thus, conclude that prioritizing exclusion over inclusion allows speakers to convey a greater number of epistemic states.

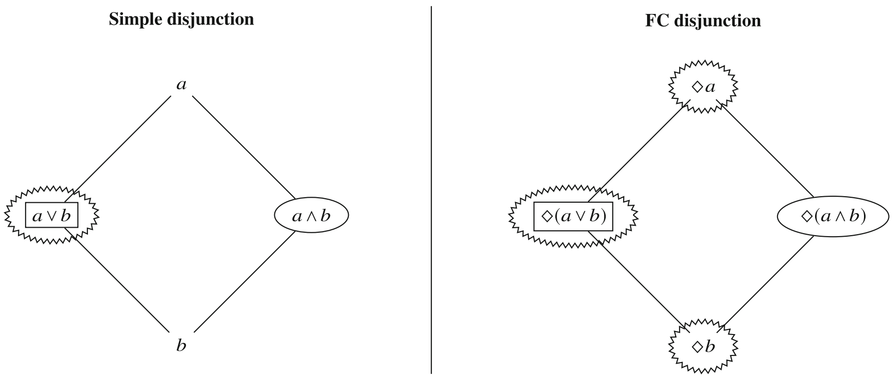


Fig. 2 Results of Innocent Exclusion and Innocent Inclusion for simple and FC disjunction. The lines represent entailment relations from right to left; the prejacent is marked with ▭, the IE alternatives with ⬭, and the II alternatives with a starburst.

the set of II alternatives is defined in parallel in (25b), with the two key differences between Innocent Exclusion and Innocent Inclusion discussed above.

(25) Given a sentence p and a set of alternatives C:

a. $IE(p, C) = \bigcap\{C' \subseteq C : C'$ is a maximal subset of C, s.t. $\{\neg q : q \in C'\} \cup \{p\}$ is consistent$\}$ [=(15b)]

b. $II(p, C) = \bigcap\{C'' \subseteq C : C''$ is a maximal subset of C, s.t. $\{r : r \in C''\} \cup \{p\} \cup \{\neg q : q \in IE(p, C)\}$ is consistent$\}$

With these definitions at hand we can write the lexical entry of $\mathcal{E}xh^{\text{IE+II}}$ in (26). Given a set of alternatives C and a prejacent p, it would assign *false* to all the IE alternatives and *true* to all the II alternatives.[19,20]

[19] An alternative one could attempt to pursue is to stick to the definition of $\mathcal{E}xh^{\text{IE}}$ and derive results parallel to those we derive using $\mathcal{E}xh^{\text{IE+II}}$ by having different assumptions than Fox (2007) about the alternatives of exhaustified constituents. For example, one might assume that the only alternatives $\mathcal{E}xh^{\text{IE}}$ projects are its sub-domain alternatives, i.e., alternatives generated by replacing the set of alternatives $\mathcal{E}xh$ operates on with its subsets. Indeed, (ia) turns out to be semantically equivalent to (ib) for all choices of p and C we checked.

(i) a. $\mathcal{E}xh^{\text{IE}}_{C'}\ [\mathcal{E}xh^{\text{IE}}_{C}\ p]$ (where $C' = \{\mathcal{E}xh^{\text{IE}}_{C''}(p) : C'' \subseteq C\}$)

b. $\mathcal{E}xh^{\text{IE+II}}_{C}\ p$

However, even if these two formulas end up being semantically equivalent in general, as we suspect, a theory of cell identification based on $\mathcal{E}xh^{\text{IE+II}}$ will allow us to capture facts that we cannot capture with $\mathcal{E}xh^{\text{IE}}$ when coupled with the specific assumption about projection in (ia) (the identity of C'). Specifically, our proposal in Sect. 4 for the semantics of *only* and the different treatment considered for IE and II alternatives in Sect. 9.1 and Bar-Lev (2018, Chap. 2) crucially rely on the distinction between IE and II alternatives, which is only available with $\mathcal{E}xh^{\text{IE+II}}$. Furthermore, distributive inferences for sentences of the form $\forall x(Px \vee Qx)$ can be derived with recursive application of $\mathcal{E}xh^{\text{IE+II}}$ along the lines of Bar-Lev and Fox (2016) (see Sect. 5.5) using Fox's assumptions about how alternatives project; recursive application of $\mathcal{E}xh^{\text{IE}}$ with the assumption about projection in (ia) won't do in this case.

[20] Note that p (the prejacent) can never be in $IE(p, C)$ and will always be in $II(p, C)$, assuming that the prejacent p must be in C and that C is finite. Namely, $p(w)$ in (i) would be redundant underthese

(26) *Innocent Exclusion+Innocent Inclusion–based exhaustivity operator*:
$$[\![\mathcal{E}xh^{\text{IE+II}}]\!](C)(p)(w) \Leftrightarrow \forall q \in IE(p, C)[\neg q(w)] \wedge \forall r \in II(p, C)[r(w)]$$

Note that $\mathcal{E}xh^{\text{IE+II}}$ has the property of cell identification in (22): it follows from its definition that the output of a single application of $\mathcal{E}xh^{\text{IE+II}}$ would be the *Cell* interpretation whenever that is not a contradiction.

(27) $\mathcal{E}xh^{\text{IE+II}}$ *derives cell identification (when possible)*:[21]
For any proposition p and any set of alternatives C:
If $Cell(p, C) \neq \bot$, then:

a. $C \setminus IE(p, C) = II(p, C)$, and (therefore)
b. $\mathcal{E}xh^{\text{IE+II}}_C(p) \Leftrightarrow Cell(p, C)$

Throughout the paper we will occasionally use the property of cell identification to justify a shortcut: whenever the *Cell* interpretation is non-contradictory, that would be what we get by applying $\mathcal{E}xh^{\text{IE+II}}$. In such cases the set of II alternatives would be identical to the set of non-IE ones.

To summarize, building on Fox's notion of Innocent Exclusion we have introduced the notion of Innocent Inclusion. We have suggested a revision of the exhaustivity operator such that it would not only assign *false* to all the IE alternatives but also assign *true* to all the II alternatives. As a conceptual motivation we have suggested that $\mathcal{E}xh$ should be able to assign a truth value to every alternative as long as this doesn't lead to a contradiction (or involves an arbitrary choice between alternatives).

3.3.4 Empirical evidence for cell identification and where to find it

At this point, though, we have not yet presented any empirical motivation to prefer $\mathcal{E}xh^{\text{IE+II}}$ over a recursive application of $\mathcal{E}xh^{\text{IE}}$; as mentioned above, for both simple disjunction and FC disjunction the view promoted here yields the same result which Fox (2007) derives by applying $\mathcal{E}xh^{\text{IE}}$ recursively. The remainder of this paper will be mainly dedicated to providing empirical evidence that cell identification in (22), repeated here, is a correct generalization, thereby arguing in favor of $\mathcal{E}xh^{\text{IE+II}}$, which derives this generalization.

Footnote 20 continued
assumptions since it would be entailed by $\forall r \in II(p, C)[r(w)]$. So (i) would be equivalent to (26), where $p(w)$ is taken out.

(i) $[\![\mathcal{E}xh^{\text{IE+II}}]\!](C)(p)(w) \Leftrightarrow p(w) \wedge \forall q \in IE(p, C)[\neg q(w)] \wedge \forall r \in II(p, C)[r(w)]$

[21] If $Cell(p, C)$ is not a contradiction, then $\{p\} \cup \{\neg q : q \in IE(p, C)\} \cup \{r : r \in C \setminus IE(p, C)\}$ is consistent. And if this is the case, there is only one maximal subset of C consistent with the prejacent and the falsity of the IE alternatives, one that contains all the non-IE alternatives. The set of II alternatives is hence the set of all non-IE alternatives: $II(p, C) = C \setminus IE(p, C)$. Since $\mathcal{E}xh^{\text{IE+II}}_C(p) \Leftrightarrow p \wedge \bigwedge\{\neg q : q \in IE(p, C)\} \wedge \bigwedge II(p, C)$, it is also true that $\mathcal{E}xh^{\text{IE+II}}_C(p) \Leftrightarrow Cell(p, C)$.

(28) *Cell identification (when possible)*:
Let S be a sentence with denotation p and C be the set of denotations of its alternatives. If $Cell(p, C)$ is not a contradiction, then S can have $Cell(p, C)$ as a strengthened meaning.

Where should we look for evidence for (28)? Note that if we take a 'standard' set of alternatives derived by replacements of only one scalar item with others, we normally derive a totally ordered set of alternatives with respect to entailment (e.g., {*some students came*, *most students came*, *all students came*}). For any prejacent with such a set of alternatives, a *Cell* interpretation would be derived by virtually any theory of scalar implicatures. (Putting aside cases of the sort discussed in Fox and Hackl 2006, all the alternatives that asymmetrically entail the prejacent would be assigned *false* and all alternatives that are entailed by the prejacent would be assigned *true*). To be more precise, as long as the set of alternatives has no subset which is 'symmetric' relative to the prejacent, it will be difficult to distinguish theories in which (28) holds from those in which it doesn't.[22]

Moreover, not every prejacent with symmetric alternatives will help us figure out whether (28) holds. What we need are cases with symmetric alternatives for which the *Cell* interpretation is non-contradictory. As we have seen in Sect. 3.2, both simple and FC disjunctions give rise to sets of alternatives that have a symmetric subset: the set $\{a, b\}$ is symmetric relative to $a \vee b$, and the same relation holds between $\{\Diamond a, \Diamond b\}$ and $\Diamond(a \vee b)$. However, only for FC disjunction is the *Cell* interpretation non-contradictory; for simple disjunction the *Cell* interpretation is contradictory and cell identification is consequently impossible. We are thus looking for prejacents having sets of alternatives with a symmetric subset which, like FC disjunction, are not closed under conjunction, so that the *Cell* interpretation may be non-contradictory. In Sects. 5–8 we will then consider more data involving disjunctions which readily give rise to symmetric alternatives and for which the *Cell* interpretation is non-contradictory. We argue that in all of those cases, the *Cell* interpretation is empirically attested, even where previous theories fail to predict it. Those data points, which have been briefly introduced in Sect. 1, will be utilized to argue for Innocent Inclusion.

Before we turn to these cases, we will deal first in Sect. 4 with an issue which immediately arises from our definition of $\mathcal{E}xh$ in (26), namely its similarity to the overt exhaustivity operator *only*. We argue for incorporating Innocent Inclusion in the semantics of *only* as well, providing an explanation for the data in (2) and maintaining a close connection between $\mathcal{E}xh$ and *only*. In Sect. 5 we motivate cell identification on the basis of the problem of universal FC brought up by Chemla (2009b), showing that Innocent Inclusion provides a global derivation of FC for structures of the schematic form $\forall x \Diamond(Px \vee Qx)$, illustrated in (3). Section 6 is concerned with a similar problem pointed out by Nouwen (2017), namely deriving FC for structures of the schematic form $\Diamond \forall x(Px \vee Qx)$ exemplified in (4). Section 7 applies Innocent Inclusion to derive simplification of disjunctive antecedents (SDA), exemplified in (5). Section 8 discusses further predictions of our account of simplification, mainly the expectation

[22] We say that a set C' is symmetric relative to a prejacent p *iff* $\forall r \in C' : p \wedge \neg r \neq \bot$ and $p \Rightarrow \bigvee C'$. We also say that an alternative q is symmetric to alternatives $q_1, \ldots, q_n$ given a prejacent p whenever the set $C' = \{q_1, \ldots, q_n\}$ isn't symmetric relative to p but $C' \cup \{q\}$ is.

that not every conditional with a disjunctive antecedent will simplify (McKay and van Inwagen 1977) and that simplification inferences will be found outside the realm of conditionals.

4 The presupposition of *only*

4.1 The connection between $\mathcal{E}xh$ and *only*

$\mathcal{E}xh$ was stated originally as a covert analog of *only*, with the minimal difference that while *only* presupposes its prejacent, $\mathcal{E}xh$ asserts it (see Fox 2007):

(29) a. $\mathcal{E}xh^{\text{IE}}$ *asserts* that its prejacent is true and also that all IE alternatives are false.
b. *Only presupposes* that its prejacent is true and asserts that all IE alternatives are false.

When we add Innocent Inclusion into the definition of $\mathcal{E}xh$ in (26), is the analogy disrupted? We claim it is not. The minimal difference can still be maintained if we assume that Innocent Inclusion is at play in the case of *only* too: what *only* presupposes is the positive part of the meaning $\mathcal{E}xh$ asserts, namely Inclusion. The analogy can then be stated as in (30): while *only* presupposes all the II alternatives, $\mathcal{E}xh$ asserts them.

(30) a. $\mathcal{E}xh^{\text{IE+II}}$ *asserts* that *all II alternatives* are true and also that all IE alternatives are false.
b. *Only presupposes* that *all II alternatives* are true and asserts that all IE alternatives are false.

We propose the lexical entry for *only* in (32) which, following (30), is only different from the entry for $\mathcal{E}xh^{\text{IE+II}}$ in (31) in presupposing rather than asserting that all II alternatives are true.[23]

(31) $$[\![\mathcal{E}xh^{\text{IE+II}}]\!](C)(p) = \lambda w.\forall r \in II(p, C)[r(w)] \wedge \forall q \in IE(p, C)[\neg q(w)]$$ [=(26)]

(32) $$[\![\text{only}]\!](C)(p) = \lambda w : \forall r \in II(p, C)[r(w)].\ \forall q \in IE(p, C)[\neg q(w)]$$

4.2 Motivation for Innocent Inclusion with *only* (Alxatib 2014)

An empirical motivation for the entry we suggested in (32) comes from work by Alxatib (2014) on the interaction between FC disjunction and *only*. Embedding FC disjunction in the scope of *only*, as in (33), yields the FC inferences in (33a) and (33b).[24]

[23] Since this is a strong presuppositional analysis, we will have to deal with arguments that *only*'s presupposition is weaker than its prejacent (Ippolito 2008). We hope that there is a way to deal with these arguments that will not destroy the picture we are trying to draw here.

[24] Example (33) might exemplify a broader generalization pertaining to all scalar implicatures under operators which are Strawson-DE but not DE, discussed in Gajewski and Sharvit (2012), Spector and Sudo (2017), Marty (2017), Anvari (2018). It is not clear to us, however, that FC disjunction gives rise to similar facts more generally, e.g., when embedded under *sorry* or *surprised*.

(33) We are only allowed to eat [ice cream or cake]$_F$.

a. $\rightsquigarrow$ We are allowed to eat ice cream.
b. $\rightsquigarrow$ We are allowed to eat cake.

Furthermore, Alxatib (2014) claims that FC inferences become presuppositions when FC disjunction is embedded in the scope of *only*. As (34) (repeated from (2)) shows, they project out of questions, as we would expect from presuppositions. The contrast between (34) and (35) shows that *only* is the culprit: in the absence of *only*, as in (35), we do not infer FC.[25]

(34) Are we only allowed to eat [ice cream or cake]$_F$?

a. $\rightsquigarrow$ We are allowed to eat ice cream.
b. $\rightsquigarrow$ We are allowed to eat cake.

(35) Are we allowed to eat ice cream or cake?

a. $\not\rightsquigarrow$ We are allowed to eat ice cream.
b. $\not\rightsquigarrow$ We are allowed to eat cake.

Given the entry for *only* in (32), the FC inferences in (33) and (34) are straightforwardly predicted to be part of the presupposition triggered by *only*. Since *only* presupposes all the II alternatives, applying *only* to the FC disjunction $\Diamond(a \vee b)$ and its set of alternatives $Alt(\Diamond(a \vee b))$ would presuppose $\Diamond a$ and $\Diamond b$, which are II, as has been established in Sect. 3. Without the entry in (32), it is not trivial to explain why the FC inferences of FC disjunction under *only* should become presuppositions.[26,27]

[25] As Chris Barker pointed out to us, a *yes* answer to (35) could lead (in certain contexts) to the inference that we are free to choose between ice cream and cake, and a *no* answer would naturally convey that we are allowed neither ice cream nor cake. Note that a similar situation arises in other cases in which a scalar implicature–generating sentence is used to form a yes/no question:

(i) Did John do some of the homework?

In this case too, a *yes* answer can lead to the inference that John did not do all of the homework, whereas a *no* answer would mean that John did not do any of the homework.

[26] Alxatib suggests two possible accounts, both relying on the assumption that there is an exhaustivity operator other than *only* in the structure, along with additional assumptions. If we are right, none of this is needed.

[27] Our entry for *only* in (32) together with our analysis of SDA in Sect. 7 predicts (i) to presuppose the disjunctive alternatives of *only*'s prejacent, in parallel to (33).

(i) Only if you work hard or inherit a fortune do you succeed.

The simplification inferences (i) gives rise to are, however, somewhat weaker than expected: (i) doesn't seem to presuppose that if you work hard you succeed, but rather the weaker presupposition that if you work hard you *might* succeed. We believe the problem is more general, since the presupposition of *only if* sentences is weaker than expected on most accounts regardless—that is, even for simple sentences that do not involve disjunction (see von Fintel 1997). We hope this can be captured with a modification of *only*'s presupposition (see in this connection fn. 23).

In parallel to our discussion of FC with *only*, we further expect simplification inferences to survive embedding in a question only in the presence of *only*. This seems to be borne out: while from (iia) we infer that if you work hard you might succeed, this is not an inference of (iib).

(ii) a. Is it true that you succeed only if you work hard or inherit a fortune?
b. Is it true that you succeed if you work hard or inherit a fortune?

Alxatib's data then provide an argument for Innocent Inclusion with *only*. We move on now to argue in favor of having Innocent Inclusion in the definition of $\mathcal{E}xh$, by providing empirical evidence for cell identification; our first argument comes from FC disjunction embedded in the scope of a universal quantifier, which we discuss in the next section.

5 The problem of universal FC

5.1 A local derivation for universal FC

Chemla (2009b) discussed sentences like (36) (repeated from (3)), where FC disjunction is embedded under universal quantification, and pointed out that they give rise to the embedded FC inferences in (36a) and (36b) (in fact, Chemla presents experimental evidence that such embedded FC inferences are as robust as in the unembedded case of (1)).

(36) Every boy is allowed to eat ice cream or cake. $\forall x \Diamond (Px \lor Qx)$

a. $\rightsquigarrow$ Every boy is allowed to eat ice cream. $\forall x \Diamond Px$

b. $\rightsquigarrow$ Every boy is allowed to eat cake. $\forall x \Diamond Qx$

One plausible analysis of the inferences in (36) (and in fact one we will be arguing in Sect. 9.2 needs to be assumed) is based on a local derivation of FC: *every boy* takes scope over an enriched FC meaning, with whatever mechanism we might have for enriching (1) (*Mary is allowed to eat ice cream or cake*) applying in the scope of *every boy*. (See Singh et al. 2016 for a possible explanation for the relative robustness of this putative local implicature.) As Chemla (2009b) points out, a local derivation is the *only* way standard implicature-based accounts can derive universal FC (see fn. 31 for the results of applying Fox's mechanism globally). In this light, (36) may just seem like yet another argument in favor of deriving implicatures at an embedded level (Cohen 1971; Landman 1998; Levinson 2000; Chierchia 2004; Chierchia et al. 2012).

5.2 Negative universal FC as an argument for a global derivation

However, as Chemla (2009b) notes, a local derivation cannot explain a very similar universal inference that arises in the negative case in (37):

(37) No student is required to solve both problem A and problem B.
$\neg\exists x \Box (Px \land Qx) \Leftrightarrow \forall x \Diamond (\neg Px \lor \neg Qx)$

a. $\rightsquigarrow$ No student is required to solve problem A. $\neg\exists x \Box Px \Leftrightarrow \forall x \Diamond \neg Px$

b. $\rightsquigarrow$ No student is required to solve problem B. $\neg\exists x \Box Qx \Leftrightarrow \forall x \Diamond \neg Qx$

The inferences from (37) to (37a) and (37b) are logically parallel to the universal FC inferences in (36). As the formulas on the right indicate, the inferences can be restated as a universal FC inference: from *Every student is allowed not to solve problem A* *or not to solve problem B* (=(37)) to *Every student is allowed not to solve problem A* (=(37a)) and *Every student is allowed not to solve problem B* (=(37b)).

An account of (37) parallel to the account of (36) is probably needed. However, a local derivation is not applicable in this case: such a derivation would require an embedded syntactic position at which the enriched FC meaning can be derived, and no such position exists in (37). Since the scope of *no student* only contains strong scalar items—*required* and *and*—no further strengthening can occur at any embedded position. Because the inferences cannot be derived from embedding the mechanism we have for (1), they must be derived at the matrix level, above negation; in other words, they necessitate a global derivation.

The lesson from Chemla (2009b) is thus that while a local derivation is sufficient for the positive case in (36), a global derivation is needed in order to explain the attested inferences of the parallel case in (37). This alone necessitates a global derivation which would presumably be applicable to both cases, given the parallelism between them. In what follows we strengthen the argument in favor of the existence of a global derivation, claiming that having only a local derivation is problematic even for the positive universal FC case in (36).

5.3 VP-ellipsis constructions as an argument for a global derivation

A local derivation faces problems for positive universal FC cases like (36) when embedded in VP ellipsis constructions where the elided material is in a DE environment, as in (38) (this argument was pointed out to us by Luka Crnič, p.c.):

(38) Every girl is allowed to eat ice cream or cake on her birthday. Interestingly, no boy is ~~allowed to eat ice cream or cake on his birthday~~. ≈

a. Every girl is allowed to eat ice cream *and* allowed to eat cake on her birthday, and $\forall x \in [\![\text{girl}]\!](\Diamond Px \wedge \Diamond Qx)$

b. no boy is allowed to eat ice cream and (likewise) no boy is allowed to eat cake on his birthday. $\neg\exists x \in [\![\text{boy}]\!]\Diamond(Px \vee Qx)$

The situation here is similar to example (7) of VP-ellipsis with unembedded FC discussed above: the first conjunct in (38) is interpreted as having an enriched FC meaning, (38a), while the elided material inside the second conjunct is interpreted as having a basic disjunctive, non-FC meaning, (38b).

At first glance this may seem unproblematic for a local derivation: we could derive the FC meaning locally for the first conjunct with an embedded $\mathcal{E}xh$, and (similarly to our analysis of (7)) this $\mathcal{E}xh$ would be absent from the elided material. Note, however, that both the antecedent and the elided material in (38) contain a bound variable, the pronoun *her* which is bound by *every girl* in the antecedent. Under the assumption that the binder of any elided bound variable in such cases has to be inside the 'parallelism domain' for ellipsis (Rooth 1992; Heim 1996), it follows that everything embedded under *every girl* has to be inside the parallelism domain.

If FC for the first sentence were derived locally, then to satisfy parallelism the elided material would be forced to have an FC meaning,[28] thus giving rise to a globally weaker meaning than the one attested for the second sentence:

(39) No boy is both allowed to eat ice cream *and* allowed to eat cake on his birthday.
$$\neg\exists x \in [\![\text{boy}]\!](\Diamond Px \wedge \Diamond Qx)$$

A global derivation of FC for the first sentence, on the other hand, would allow us to satisfy parallelism without generating embedded FC meaning for the second sentence. Having $\mathcal{E}xh$ above the binder of *her* allows for a parallelism domain containing the binder of *her* while not containing $\mathcal{E}xh$.[29]

5.4 Deriving universal FC globally

As we have seen, a local derivation turns out to be insufficient for universal FC, in positive and negative cases alike, and a global derivation is needed. We want to show now that once we assume Innocent Inclusion, such a derivation is indeed available. Before we proceed to do that, a clarification is in order. Our arguments above do not by any means show that local derivations aren't available for positive cases where an embedded syntactic position for $\mathcal{E}xh$ derives universal FC. All they show is that local derivations of universal FC are insufficient, and that there must also be a way to derive universal FC globally. We have no intention to claim that local derivations aren't available, and no reason to do so given the grammatical theory we assume; in Sect. 9.2 we will in fact utilize local derivations for cases very similar to universal FC for which a global derivation isn't sufficient (see also fn. 33). For the time being, though, we will focus on global derivations.

Consider the set of alternatives we generate for (36), in (41). We assume that alternatives where the universal quantifier *every* is replaced with the existential quantifier *some* are generated (this assumption raises various questions to which we'll return in Sect. 5.5). Consequently, the set of alternatives is multiplied by 2 compared to the four alternatives of unembedded FC disjunction in (11b); we end up with the eight alternatives in (41).

(40) Every boy is allowed to eat ice cream or cake. [=(36)] $\forall x \Diamond (Px \vee Qx)$

(41) *Set of alternatives for universal FC*:
$$Alt(\forall x \Diamond (Px \vee Qx)) =$$
$$\{\underbrace{\forall x \Diamond (Px \vee Qx)}_{\text{Prejacent}}, \underbrace{\forall x \Diamond Px\ ,\ \forall x \Diamond Qx}_{\text{Universal-disjunctive alts.}}, \underbrace{\forall x \Diamond (Px \wedge Qx)}_{\text{Universal-conjunctive alt.}},$$
$$\underbrace{\exists x \Diamond (Px \vee Qx)}_{\text{Existential alt.}}, \underbrace{\exists x \Diamond Px\ ,\ \exists x \Diamond Qx}_{\text{Existential-disjunctive alts.}}, \underbrace{\exists x \Diamond (Px \wedge Qx)}_{\text{Existential-conjunctive alt.}}\}$$

[28] See Crnič (2015) for arguments showing that truly embedded $\mathcal{E}xh$ is taken into account for parallelism considerations.

[29] In fact, even if the parallelism domain contained $\mathcal{E}xh$, the correct result would still be derived under a global derivation of universal FC (when $\mathcal{E}xh$ has scope over the binder), since $\mathcal{E}xh$ could have scope above *no* in the second sentence.

The universal FC inference follows straightforwardly with one application of $\mathcal{E}xh^{\text{IE+II}}$, since $\forall x \Diamond Px$ and $\forall x \Diamond Qx$ are II. Let us see this in greater detail.

In order to determine which alternatives are II, we first have to determine which are IE. The maximal sets of alternatives that can be assigned *false* consistently with the prejacent are in (42a), and their intersection, which is the set of IE alternatives, is in (42b). The IE alternatives are then $\forall x \Diamond(Px \wedge Qx)$ and $\exists x \Diamond(Px \wedge Qx)$.[30]

(42) a. Maximal sets of alternatives in $Alt(\forall x \Diamond(Px \vee Qx))$ that can be assigned *false* consistently with $\forall x \Diamond(Px \vee Qx)$:
(i) $\{\forall x \Diamond Px, \forall x \Diamond Qx, \forall x \Diamond(Px \wedge Qx), \exists x \Diamond(Px \wedge Qx)\}$
(ii) $\{\forall x \Diamond Px, \exists x \Diamond Px, \forall x \Diamond(Px \wedge Qx), \exists x \Diamond(Px \wedge Qx)\}$
(iii) $\{\forall x \Diamond Qx, \exists x \Diamond Qx, \forall x \Diamond(Px \wedge Qx), \exists x \Diamond(Px \wedge Qx)\}$

b. $IE(\forall x \Diamond(Px \vee Qx), Alt(\forall x \Diamond(Px \vee Qx)))) = \bigcap$ (42a) $=$
$\{\forall x \Diamond(Px \wedge Qx), \exists x \Diamond(Px \wedge Qx)\}$

If we were to apply Fox's (2007) $\mathcal{E}xh^{\text{IE}}$ recursively in this case, we would only derive the weak inferences $\exists x \Diamond Px$ and $\exists x \Diamond Qx$, but would fail to derive the stronger $\forall x \Diamond Px$ and $\forall x \Diamond Qx$.[31]

Let us now see that, in contrast, the latter are II alternatives and hence derived as inferences by $\mathcal{E}xh^{\text{IE+II}}$, which in this case determines the truth value of every alternative. The case of universal FC thus illustrates our claim from Sect. 3.3.2, that $\mathcal{E}xh^{\text{IE}}$ doesn't predict cell identification in (22), and moreover that cell identification seems to hold empirically.

To know what alternatives are II we should check what are the maximal sets of alternatives that can be assigned *true* consistently with the prejacent and the falsity of all IE alternatives. Namely, what are the maximal sets of alternatives that are consistent with the truth of the prejacent $\forall x \Diamond(Px \vee Qx)$ taken together with the falsity of $\exists x \Diamond(Px \wedge Qx)$? (We can ignore the other IE alternative, $\forall x \Diamond(Px \wedge Qx)$, since its

[30] As in the case of unembedded FC disjunction (see fn. 13), the following derivation of universal FC does not depend on the exclusion of any of the IE alternatives. In many cases they would not be relevant and thus would not be assigned *false*.

[31] This is so since the set of exhaustified alternatives for the second level of exhaustification is as follows:

(i) $Alt(\mathcal{E}xh^{\text{IE}}(\forall x \Diamond(Px \vee Qx))) =$
$\{\mathcal{E}xh^{\text{IE}}(\forall x \Diamond(Px \vee Qx)) = \forall x \Diamond(Px \vee Qx) \wedge \neg\exists x \Diamond(Px \wedge Qx),$
$\mathcal{E}xh^{\text{IE}}(\forall x \Diamond Px) = \forall x \Diamond Px \wedge \neg\exists x \Diamond Qx,$
$\mathcal{E}xh^{\text{IE}}(\forall x \Diamond Qx) = \forall x \Diamond Qx \wedge \neg\exists x \Diamond Px,$
$\mathcal{E}xh^{\text{IE}}(\forall x \Diamond(Px \wedge Qx)) = \forall x \Diamond(Px \wedge Qx),$
$\mathcal{E}xh^{\text{IE}}(\exists x \Diamond(Px \vee Qx)) = \exists x \Diamond(Px \vee Qx) \wedge \neg\exists x \Diamond(Px \wedge Qx) \wedge \neg\forall x \Diamond(Px \vee Qx),$
$\mathcal{E}xh^{\text{IE}}(\exists x \Diamond Px) = \exists x \Diamond Px \wedge \neg\forall x \Diamond Px \wedge \neg\exists x \Diamond Qx,$
$\mathcal{E}xh^{\text{IE}}(\exists x \Diamond Qx) = \exists x \Diamond Qx \wedge \neg\forall x \Diamond Qx \wedge \neg\exists x \Diamond Px,$
$\mathcal{E}xh^{\text{IE}}(\exists x \Diamond(Px \wedge Qx)) = \exists x \Diamond(Px \wedge Qx) \wedge \neg\forall x \Diamond(Px \vee Qx)\}$

The last five alternatives contradict the prejacent and hence can be trivially excluded. The only non-trivially IE alternatives are $\mathcal{E}xh^{\text{IE}}(\forall x \Diamond Px)$ and $\mathcal{E}xh^{\text{IE}}(\forall x \Diamond Qx)$; the negation of both yields $(\forall x \Diamond Px \rightarrow \exists x \Diamond Qx) \wedge (\forall x \Diamond Qx \rightarrow \exists x \Diamond Px)$. Taken together with the prejacent, this yields the result of the second application of $\mathcal{E}xh^{\text{IE}}$ in (ii):

(ii) $\forall x \Diamond(Px \vee Qx) \wedge \neg\exists x \Diamond(Px \wedge Qx) \wedge \exists x \Diamond Px \wedge \exists x \Diamond Qx$

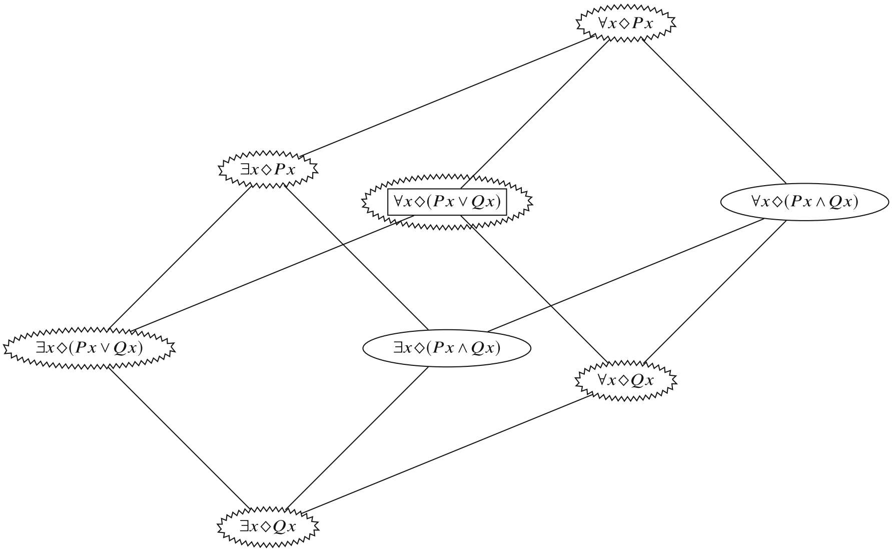


Fig. 3 Results of Innocent Exclusion and Innocent Inclusion for universal FC (notation as in Fig. 2)

falsity is entailed by the falsity of $\exists x \Diamond (Px \wedge Qx)$.) As in the case of unembedded FC disjunction, there is only one such set since all the non-IE alternatives together are consistent with the prejacent and the falsity of all IE alternatives, as in (43a). Therefore the set of II alternatives in (43b) contains all the non-IE alternatives.

(43) a. Maximal sets of alternatives in $Alt(\forall x \Diamond (Px \vee Qx))$ that can be assigned *true* consistently with $\forall x \Diamond (Px \vee Qx) \wedge \neg \exists x \Diamond (Px \wedge Qx)$:

(i) $\{\forall x \Diamond (Px \vee Qx), \forall x \Diamond Px, \forall x \Diamond Qx,$
$\exists x \Diamond (Px \vee Qx), \exists x \Diamond Px, \exists x \Diamond Qx\}$

b. $II(\forall x \Diamond (Px \vee Qx), Alt(\forall x \Diamond (Px \vee Qx)))) = \bigcap$ (43a) $=$
$\{\forall x \Diamond (Px \vee Qx), \forall x \Diamond Px, \forall x \Diamond Qx, \exists x \Diamond (Px \vee Qx), \exists x \Diamond Px, \exists x \Diamond Qx\}$

As in the case of unembedded FC disjunction, exhaustification here assigns a truth value to every alternative. Most importantly, the alternatives $\forall x \Diamond Px$ and $\forall x \Diamond Qx$ are members in the set of II alternatives. Applying $\mathcal{E}xh^{\text{IE+II}}$ would then assign them *true* and derive the desired universal FC inferences, as in (44).

(44) $\mathcal{E}xh^{\text{IE+II}}_{Alt(\forall x(\Diamond(Px \vee Qx))} \forall x \Diamond (Px \vee Qx)$
$\Leftrightarrow \forall x \Diamond Px \wedge \forall x \Diamond Qx \wedge \neg \exists x \Diamond (Px \wedge Qx)$

In fact, once we have computed the IE alternatives, we could conclude the result in (44) without explicitly computing what is II: since the *Cell* interpretation for $\forall x \Diamond (Px \vee Qx)$ is not contradictory, by cell identification (in (27)) it follows that applying $\mathcal{E}xh^{\text{IE+II}}$ yields the *Cell* interpretation, as in (44). The results of Innocent Exclusion and Innocent Inclusion are represented in Fig. 3.

Recall our motivation for a global account of universal FC based on the negative universal FC case in (37). Since the same kind of entailment relations hold between the alternatives in the negative case (37) and the positive case (36) (assuming they give rise to parallel alternatives, as in (46)),[32] the result is parallel, as can be seen in (47).[33]

(45) No student is required to solve both problem A and problem B. [=(37)]
$\neg\exists x\Box(Px \wedge Qx)$

(46) $Alt(\neg\exists x\Box(Px \wedge Qx)) =$
$\{\neg\exists x\Box(Px \wedge Qx), \neg\exists x\Box Px, \neg\exists x\Box Qx, \neg\exists x\Box(Px \vee Qx),$
$\neg\forall x\Box(Px \wedge Qx), \neg\forall x\Box Px, \neg\forall x\Box Qx, \neg\forall x\Box(Px \vee Qx)\}$

(47) $\mathcal{E}xh^{\text{IE+II}}_{Alt(\neg\exists x(\Box(Px\wedge Qx))}\neg\exists x\Box(Px \wedge Qx)$
$\Leftrightarrow \neg\exists x\Box Px \wedge \neg\exists x\Box Qx \wedge \forall x\Box(Px \vee Qx)$

5.5 A remark on multiple replacements

Before we move on to another piece of evidence for Innocent Inclusion, we'd like to elaborate on an assumption we made which may seem problematic. We assumed that whenever there are two scalar items in a sentence, alternatives can be generated by replacing both items with their alternatives. This assumption has been argued by Fox (2007, fn. 35, attributing this point to a conversation with Gennaro Chierchia) to be problematic for cases which are very similar in their structure to the case of $\forall x\Diamond(Px \vee Qx)$ discussed in this section, namely sentences of the form $\forall x(Px \vee Qx)$, as in (48). Such sentences have so-called distributive inferences, as in (48a) and (48b), which are standardly thought to be derived by negating the universal disjunctive alternatives $\forall x Px$ and $\forall x Qx$.

(48) Every one of these people is singing or dancing. $\forall x(Px \vee Qx)$
a. $\rightsquigarrow$ At least one of these people is singing. $\exists x Px$
b. $\rightsquigarrow$ At least one of these people is dancing. $\exists x Qx$

[32] A reviewer asks what justifies taking *not every* to be an alternative to *no*, a crucial assumption for deriving parallel alternatives in the positive and negative cases of universal FC. First, we would like to point out that this is needed in order to explain how the scalar implicature in (ia) comes about for (i).

(i) If you do nothing you are told, your parents will definitely be upset.
a. $\rightsquigarrow$ It's not the case that if you fail to do everything you are told, your parents will definitely be upset.

Second, *not every* is expected to be an alternative to *no* if we combine the structural view of alternative generation in Katzir (2007) and Fox and Katzir (2011), together with a decompositional account of *no* assumed in much current literature (according to which it's syntactically composed of negation and an existential quantifier; see, e.g., Sauerland 2000).

[33] Chemla's (2009b) results show a significant difference in robustness between the universal FC inferences in (36) and (37). This might follow from the existence of another route to embedded FC in the positive case of universal FC, namely that of a local derivation of FC, which is unavailable in the negative case. The presence of negation might also play a role here by potentially introducing more alternatives; the universal FC inferences will not be derived if, as we propose in Sect. 8.2, alternatives where negation is replaced with $\mathcal{E}xh$ are generated.

The problem is that assuming multiple replacements makes the universal disjunctive alternatives non-IE: the alternative $\exists x Qx$ (derived by multiple replacements) is symmetric to $\forall x Px$, and of course the same goes for $\exists x Px$ and $\forall x Qx$. We should add that none of them is II either, so it doesn't matter whether we apply $\mathcal{E}xh^{\text{IE}}$ or $\mathcal{E}xh^{\text{IE+II}}$. In either case we do not derive the distributive inferences assuming multiple replacements.

We would like to turn the argument on its head based on two recent observations. First, Crnič et al. (2015) have argued that there has to be a way to derive distributive inferences without the negation of the universal disjunctive alternatives, because at least sometimes the former are not accompanied by the latter (in fact, they found no evidence for the existence of the latter). Second, Bar-Lev and Fox (2016) have shown that while Fox's (2007) objection to multiple replacements holds if we only apply $\mathcal{E}xh^{\text{IE}}$ once, it doesn't hold if we apply $\mathcal{E}xh^{\text{IE}}$ recursively, in which case we do derive distributive inferences even if we have multiple replacements (importantly for our purposes, the same result holds also for $\mathcal{E}xh^{\text{IE+II}}$). Moreover, the results of this derivation are distributive inferences without the universal disjunctive alternatives being negated, which is precisely what has been argued by Crnič et al. to be needed. So at the end of the day, the assumption of multiple replacements ends up not only unproblematic for deriving distributive inferences, but in fact superior to its denial in providing a way to account for Crnič et al.'s data.

However, the situation is more involved. As Crnič et al. (2015) point out, distributive inferences for sentences of the parallel form $\Box(p \vee q)$, as in (49), do seem to be accompanied by the negation of the universal disjunctive alternatives, as in (49a,b):

(49) You are required to solve problem A or problem B. $\Box(p \vee q)$

a. $\rightsquigarrow$ You are not required to solve problem A. $\neg\Box p$

b. $\rightsquigarrow$ You are not required to solve problem B. $\neg\Box q$

In this case, then, the assumption of multiple replacements seems problematic again for the reason brought up by Fox (2007): due to the symmetry caused by multiple replacements, the universal disjunctive alternatives would not be IE. But in this case we in fact want them to be IE. A way to describe the facts would be to say that with $\Box(p \vee q)$ we do not generate alternatives by replacing $\Box$ with its alternatives (at least not necessarily), while with $\forall x(Px \vee Qx)$ we do generate alternatives by replacing $\forall$ with its alternatives. As a result, there would be no symmetry in the case of $\Box(p \vee q)$ to begin with, and the universal disjunctive alternatives would end up IE.

While this may seem outrageously stipulative, there is in fact independent evidence for distinguishing between quantificational DPs and modals in exactly this way. Chierchia (2013, Chaps. 4.2 and 7) discusses the fact that universal quantifiers generate intervention effects on the licensing of NPIs, but only if the quantifier at stake is a quantificational DP, as in (50a); modals do not intervene, as shown in (50b):

(50) a. *I doubt that everyone ever solved that problem.

b. I doubt that John must ever smile (as part of his job).

(Chierchia 2013, p. 201)

Chierchia's account of the badness of (50a), very roughly speaking, relies on the idea that it is impossible for the alternatives generated by *ever* to be "active" without the alternatives generated by replacing *every* with its alternatives being "active" as well whenever *every* intervenes between the exhaustivity operator (the licensor of NPIs in his theory) and *ever*. To explain why (50b) is fine, he proposes that in a parallel situation with *must* instead of *every* it is possible for the alternatives of *must* to be inactive. For an explicit proposal see Chierchia (2013, Chap. 7).[34] For further reasons to adopt the assumption of multiple replacements, as well as some problems and solutions, we refer the interested reader to Magri (2009), Chemla (2009b), Chemla and Spector (2011), Romoli (2012), Trinh and Haida (2015), Gotzner and Romoli (2017).

6 Free choice for sentences of the form $\Diamond\forall x(Px \vee Qx)$ (Nouwen 2017)

A similar problem to that of universal FC has been discussed in Nouwen (2017). While the issue with universal FC is that of deriving from $\forall x\Diamond(Px \vee Qx)$ the inferences $\forall x\Diamond Px$ and $\forall x\Diamond Qx$, Nouwen's (2017) concern is with deriving the analogous inferences for sentences where an existential modal takes scope above a universal quantifier; namely deriving from sentences of the form $\Diamond\forall x(Px \vee Qx)$ the inferences $\Diamond\forall x Px$ and $\Diamond\forall x Qx$.

Providing direct evidence for this is however more difficult than might seem at first, due to the ability of disjunction to take non-surface scope. One could think that (51) is such a case:

(51) John allowed every kid to eat ice cream or cake. Possibly: $\Diamond\forall x(Px \vee Qx)$
  a. $\rightsquigarrow$ John allowed every kid to eat ice cream. $\Diamond\forall x Px$
  b. $\rightsquigarrow$ John allowed every kid to eat cake. $\Diamond\forall x Qx$

[34] An extremely interesting data point, which we think is related to this issue, was brought to our attention by an anonymous reviewer:

(i) a. Q: What do I have to do in order to get credit for the course?
  b. A: You may either write an essay or give a presentation.

As the reviewer points out, (i) seems to show that a sentence of the form $\Diamond(p \vee q)$ should not have the alternative $\Box(p \vee q)$; for if $\Box(p \vee q)$ was an alternative to $\Diamond(p \vee q)$, the question in (ia) would make it relevant and hence excluded, and we would expect the inference that you *don't* have to write an essay or give a presentation in order to get credit. In fact, we get the opposite inference, that you do have to do one of them. This data point lends further support to the idea that the alternatives of intervening modals don't have to be generated to begin with. In the absence of the alternative $\Box(p \vee q)$, the observed inference could be derived by considering alternatives like *You may take an exam (to get credit for the course)*, etc., and negating all of them, to the effect that nothing other than writing an essay and giving a presentation would let you get credit for the course.

Comparing (i) with the following case, which features a quantificational DP instead of a modal, is particularly telling: Here we do not infer $\forall x(Px \vee Qx)$ but rather its negation, namely that not every boy solved problem A or B (and consequently the intuition that the question wasn't fully answered). This is explained by assuming that in this case the alternatives of *some boys* must be generated.

(ii) a. Q: What did every boy do?
  b. A: Some boys solved problem A or problem B.

However, as Nouwen points out, it is difficult to rule out other scope possibilities for (51). Besides the scope construal we are after, *allowed* $>$ *every* $>$ *or*, there are two other possible LFs which are expected to derive the desired inferences: *every* $>$ *allowed* $>$ *or*, which is essentially a universal FC construal, and *allowed* $>$ *or* $>$ *every*, which would make it a basic FC disjunction construction.

How can we make sure that we are dealing with an underlying structure of the intended form? Here is an attempt. First, let us make sure we use a structure in which the existential modal takes scope above *every*; (52) seems to only admit such a reading:

(52) The teacher is OK with every student talking now.

Based on (52), we take it that the existential modal in (53) (repeated from (4)) takes scope above *every*; and following Larson's (1985) observation that dislocated *either* fixes the scope of disjunction, we assume that disjunction here takes scope below *every*.

(53) The teacher is OK with every student either talking to Mary or to Sue.
a. $\rightsquigarrow$ The teacher is OK with every student talking to Mary.
b. $\rightsquigarrow$ The teacher is OK with every student talking to Sue.

We have then an example of the form $\Diamond\forall x(Px \vee Qx)$ which indeed admits the inferences $\Diamond\forall x Px$ and $\Diamond\forall x Qx$.[35] Given the difficulty in providing direct evidence for FC with $\Diamond\forall x(Px \vee Qx)$ illustrated above, Nouwen's evidence comes instead from FC with ability modals. Since the connection between FC with ability modals and (53) isn't necessarily transparent, and since it turns out to involve some complications for our account, we'll delay its discussion until Sect. 6.2 and focus for now on deriving FC for sentences like (53).

6.1 Deriving FC for $\Diamond\forall x(Px \vee Qx)$

We have seen empirical evidence that a sentence with the form $\Diamond\forall x(Px \vee Qx)$ should be strengthened to entail $\Diamond\forall x Px$ and $\Diamond\forall x Qx$. We will see now that this follows from Innocent Inclusion.

Note, first, that these entailments will not follow from exhaustification at an embedded level, since strengthening of $\forall x(Px \vee Qx)$ could only give us irrelevant inferences (see our discussion of sentences of this form in Sect. 5.5). The desired result would have been derived by embedded exhaustification if we could strengthen $\forall x(Px \vee Qx)$ to $\forall x(Px \wedge Qx)$, but that of course never happens: (54a) and (54b) cannot be taken to follow from (54).

[35] As pointed out to us by Gennaro Chierchia, (i) can be taken to further support Nouwen's descriptive claim. Specifically, if we assume that *any* is an existential quantifier, it must be strengthened to a universal quantifier in this environment, and this can be done by exhaustification, as pointed out in Chierchia (2013), Crnič (2017).

(i) a. The teacher allowed every girl to talk to any of her friends.
b. The teacher is OK with every girl talking to any of her friends.

(54) Every kid ate ice cream or cake.

a. $\not\rightsquigarrow$ Every kid ate ice cream.

b. $\not\rightsquigarrow$ Every kid ate cake.

We would therefore like the required strengthening to follow from global exhaustification. Nouwen claims that Fox (2007) incorrectly predicts the disjunctive alternatives $\Diamond\forall x Px$ and $\Diamond\forall x Qx$ to be IE and, therefore, that their falsity would be incorrectly derived. But this holds only as long as we ignore the alternatives $\Diamond\exists x Px$ and $\Diamond\exists x Qx$. Admitting the latter alternatives makes the former non-IE. Suppose then that we derive the following set of alternatives:

(55) $Alt(\Diamond\forall x(Px \vee Qx)) =$
$$\{\underbrace{\Diamond\forall x(Px \vee Qx)}_{\text{Prejacent}}, \underbrace{\Diamond\forall x Px\ ,\ \Diamond\forall x Qx}_{\text{Universal-disjunctive alts.}}, \underbrace{\Diamond\forall x(Px \wedge Qx)}_{\text{Universal-conjunctive alt.}},$$
$$\underbrace{\Diamond\exists x(Px \vee Qx)}_{\text{Existential alt.}}, \underbrace{\Diamond\exists x Px\ ,\ \Diamond\exists x Qx}_{\text{Existential-disjunctive alts.}}, \underbrace{\Diamond\exists x(Px \wedge Qx)}_{\text{Existential-conjunctive alt.}}\}$$

As desired, the disjunctive alternatives are not IE, due to the presence of the existential alternatives.

(56) a. Maximal sets of alternatives in $Alt(\Diamond\forall x(Px \vee Qx))$ that can be assigned *false* consistently with $\Diamond\forall x(Px \vee Qx)$:

(i) $\{\Diamond\forall x Px, \Diamond\forall x Qx, \Diamond\forall x(Px \wedge Qx), \Diamond\exists x(Px \wedge Qx)\}$

(ii) $\{\Diamond\forall x Px, \Diamond\exists x Px, \Diamond\forall x(Px \wedge Qx), \Diamond\exists x(Px \wedge Qx)\}$

(iii) $\{\Diamond\forall x Qx, \Diamond\exists x Qx, \Diamond\forall x(Px \wedge Qx), \Diamond\exists x(Px \wedge Qx)\}$

b. $IE(\Diamond\forall x(Px \vee Qx), Alt(\Diamond\forall x(Px \vee Qx)))) = \bigcap$ (56a) $=$
$$\{\Diamond\forall x(Px \wedge Qx), \Diamond\exists x(Px \wedge Qx)\}$$

Once again, the recursive application of $\mathcal{E}xh^{\text{IE}}$ falls short of deriving the desired inferences and derives instead the weaker inferences $\Diamond\exists x Px$ and $\Diamond\exists x Qx$ (for essentially the same reasons that it failed in the universal FC case in (36), where the universal quantifier had wide scope).[36]

With Innocent Inclusion, on the other hand, we derive the right result. As might be obvious at this point, the derivation is completely parallel to that of universal FC, and so is the output of $\mathcal{E}xh^{\text{IE+II}}$: since the *Cell* interpretation is non-contradictory, applying $\mathcal{E}xh^{\text{IE+II}}$ yields that interpretation, i.e., the falsity of all the IE alternatives and the truth of all the non-IE ones (which, recall, are II whenever the *Cell* interpretation is consistent):

(57) $\mathcal{E}xh^{\text{IE+II}}_{Alt(\Diamond\forall x(Px \vee Qx))}\Diamond\forall x(Px \vee Qx)$
$\Leftrightarrow \Diamond\forall x Px \wedge \Diamond\forall x Qx \wedge \neg\Diamond\exists x(Px \wedge Qx)$

To conclude, we have shown that by assuming Innocent Inclusion we can account for FC with sentences of the form $\Diamond\forall x(Px \vee Qx)$, which, as Nouwen (2017) pointed out,

[36] Since the entailment relations between the alternatives are the same in the cases of $\forall x\Diamond(Px \vee Qx)$ and $\Diamond\forall x(Px \vee Qx)$, the computation is the same as in fn. 31 modulo the relative scope of $\Diamond$ and $\forall x$.

is problematic for previous implicature approaches to FC.[37] In Sect. 7 we move on to argue that Innocent Inclusion accounts for simplification of disjunctive antecedents. Before we do that, we still owe a discussion of FC with ability modals, which is what Nouwen provided as evidence that we need FC inferences for sentences of the form $\Diamond\forall x(Px \vee Qx)$, and for which our account does not fare as well as for the more transparent case we discussed so far.

6.2 FC with ability modals

It has been claimed that ability modals like *can* are a combination of both existential and universal quantifiers, in the following sense (see Nouwen 2017, ex. (11)–(12) and references):

(58) x can do A *iff there is* an action available to x that would *reliably* bring about A.

Assuming this analysis, (59) gives rise to the meaning in (59a); as (59b) illustrates, this is the kind of logical structure we are concerned with.

(59) Betty can balance a fishing rod on her nose or on her chin. (Geurts 2010)
  a. There is a proposition p (characterizing an action by Betty) such that in all worlds[38] where p is true, either Betty balances a fishing rod on her nose or on her chin.
  b. $\exists p(\forall w \in p(Pw \vee Qw))$

As Geurts (2010) observes, (59) has an FC inference. Given the analysis sketched above for the semantics of *can*, we get the inference pattern exemplified in (60) with the formulas on the right, which is the inference pattern we are after:

(60) Betty can balance a fishing rod on her nose or on her chin. $\exists p(\forall w \in p(Pw \vee Qw))$
  a. $\rightsquigarrow$ Betty can balance a fishing rod on her nose. $\exists p(\forall w \in p(Pw))$
  b. $\rightsquigarrow$ Betty can balance a fishing rod on her chin. $\exists p(\forall w \in p(Qw))$

[37] Nouwen's more general claim is that standard implicature-based analyses of FC rely on *distribution over disjunction* as a necessary condition for deriving FC. Within such approaches, $\phi(a) \wedge \phi(b)$ can only be derived from $\phi(a \vee b)$ if ϕ distributes over disjunction.

(i) *Distribution over disjunction*: ϕ distributes over disjunction *iff* $\phi(a \vee b) \Leftrightarrow \phi(a) \vee \phi(b)$.

However, the context surrounding disjunction does not distribute over disjunction in both universal FC, (iia), and the Nouwen (2017) case discussed in this section, (iib):

(ii) a. $\forall x\Diamond(Px \vee Qx) \not\Leftrightarrow (\forall x\Diamond Px) \vee (\forall x\Diamond Qx)$
  b. $\Diamond\forall x(Px \vee Qx) \not\Leftrightarrow (\Diamond\forall x Px) \vee (\Diamond\forall x Qx)$

As we have shown, Innocent Inclusion derives FC for both cases. In other words, our analysis does not depend on distribution over disjunction for the derivation of FC inferences.

[38] Of course, the set of worlds quantified over here and in what follows is not the set of all possible worlds but rather a subset thereof, determined, e.g., by a modal base and an ordering source.

Our account of FC with $\Diamond\forall x(Px \vee Qx)$ would extend to this case only if we could assume that a sentence with ability modals that can be written as $\exists p(\forall w \in p(Qw \vee Qw))$ has the alternatives determined by that syntactic form (so that we would have alternatives such as $\exists p(\exists w \in p(Qw)))$. In order to extend our account to such cases, we will need to say that there is a covert universal quantifier in the structure, which can be replaced with an existential one.[39] Since we cannot defend this assumption at this stage, we have to leave FC with ability modals as an open problem.

7 Simplification of disjunctive antecedents (SDA)

7.1 The puzzle of SDA as an FC puzzle

Conditionals with disjunctive antecedents usually lead to the inference that the disjunctive alternatives are true, as in (61) (repeated from (5)). This is known as simplification of disjunctive antecedents (SDA):

(61) If you eat ice cream or cake, you will feel guilty. $(p \vee q) \Box\!\!\rightarrow r$

a. $\rightsquigarrow$ If you eat ice cream, you will feel guilty. $p \Box\!\!\rightarrow r$

b. $\rightsquigarrow$ If you eat cake, you will feel guilty. $q \Box\!\!\rightarrow r$

Whether SDA should be semantically valid or not has been a topic of much debate. While under a strict-conditional analysis SDA is valid, it isn't valid within a variably strict semantics for conditionals (Stalnaker 1968; Lewis 1973):[40]

(62) *Variably strict conditional*:
$p \Box\!\!\rightarrow q$ is true *iff* the closest p-worlds are q-worlds.

Fine (1975) and Nute (1975) have argued against a variably strict analysis on these grounds (namely, its inability to derive what seems to be a valid inference). But as Fine (1975) notes, accepting SDA as semantically valid (as in (63a)) together with the assumption that truth-conditionally equivalent formulas can be substituted without affecting the truth conditions ('substitutivity') leads to the unwelcome validity of (63b) (since p is equivalent to $p \vee (p \wedge q)$):

(63) a. Semantic validation of SDA: $(p \vee q) \Box\!\!\rightarrow r \models p \Box\!\!\rightarrow r$

b. $p \Box\!\!\rightarrow r \models (p \wedge q) \Box\!\!\rightarrow r$

Loewer (1976) was probably the first to suggest an analogy with FC disjunction.[41] Loewer points out that SDA should not follow from the basic semantics, for a reason

39 Note though that the presence of a universal quantifier in the structure might suffice, since disjunction could take scope above it (just as Nouwen pointed out for (51)).

40 We remain agnostic regarding the proper variably strict analysis. In what follows we occasionally refer to "the closest world," as if we assumed a Stalnakerian analysis, but this is only done for ease of presentation. As the reader may verify, the results in this section hold for a Lewisian system as well.

41 Loewer (1976) writes: "Notice the similarity between the two situations. In both cases the surface form of an English sentence is 'Modal operator (A or B)', but its logical form seems to be 'Modal operator A and modal operator B'" (p. 534).

parallel to the following reason why FC should not follow from the basic semantics (a closely connected problematization of FC is found in Kamp 1974): assuming (64a) together with substitutivity would lead to (64b):

(64) a. Semantic validation of FC: $\Diamond(p \vee q) \models \Diamond p$
b. $\Diamond p \models \Diamond(p \wedge q)$

While for both FC and SDA substitutivity can be (and has been) rethought, the implicature account of FC provides a simple way out. In what follows we demonstrate that $\mathcal{E}xh^{\text{IE+II}}$ provides a simple way out in the case of SDA as well.

7.2 Deriving SDA with Innocent Inclusion

The set of alternatives we derive for the basic SDA example in (61), which is of the form $(p \vee q) \,\Box\!\!\rightarrow r$, is the set of alternatives derived by replacing disjunction with the disjuncts and conjunction, assuming that the conditional itself does not generate any other alternatives.

(65) $Alt((p \vee q) \,\Box\!\!\rightarrow r) = \{\underbrace{(p \vee q) \,\Box\!\!\rightarrow r}_{\text{Prejacent}}, \underbrace{p \,\Box\!\!\rightarrow r, q \,\Box\!\!\rightarrow r}_{\text{Disjunctive alts.}}, \underbrace{(p \wedge q) \,\Box\!\!\rightarrow r}_{\text{Conjunctive alt.}}\}$

Note that, just like in the basic FC case, the conjunction of the disjunctive alternatives, $(p \,\Box\!\!\rightarrow r) \wedge (q \,\Box\!\!\rightarrow r)$, is absent from the set of alternatives, i.e., the set is not closed under conjunction.

Let us see now what is IE given the truth conditions in (62). What's important here is (a) that, as in the case of FC and simple disjunction, the truth of the prejacent is consistent with the falsity of one disjunctive alternative, and (b) that it is inconsistent with the falsity of both disjunctive alternatives (since, as we will see, it entails the disjunction of the disjunctive alternatives). Take for example the alternative $p \,\Box\!\!\rightarrow r$. The truth of the prejacent alone doesn't ensure the truth of this alternative since it is possible that the closest $p \vee q$-world is a $q \wedge \neg p$ world that satisfies r, and the closest p world is a $\neg r$ world. But if we take the prejacent to be true and $q \,\Box\!\!\rightarrow r$ to be false, then $p \,\Box\!\!\rightarrow r$ cannot be false anymore (the closest $p \vee q$-world is either the closest p-world or the closest q-world). For this reason, $p \,\Box\!\!\rightarrow r$ is not IE, and obviously the same holds for $q \,\Box\!\!\rightarrow r$. Just as in the case of FC disjunction, the only IE alternative then is the conjunctive alternative.[42]

[42] The excludability of the conjunctive alternative, which is shared between our account and Franke's, might seem objectionable since the falsity of $(p \wedge q) \,\Box\!\!\rightarrow r$ does not seem to be a necessary implicature of $(p \vee q) \,\Box\!\!\rightarrow r$. Note first that the falsity of $(p \wedge q) \,\Box\!\!\rightarrow r$ is at least consistent with $(p \vee q) \,\Box\!\!\rightarrow r$:

(i) If you now drink a bottle of beer or a shot of whisky you'll feel great, but if you drink both you'll feel really bad.

Recall moreover that a similar objection was discussed in fn. 13 (and fn. 30) regarding the excludability of the conjunctive alternative in FC disjunction. Our response there applies here as well: since the set of alternatives is not closed under conjunction and the conjunctive alternative is not the conjunction of the disjunctive ones, it is possible for the disjunctive alternatives to be relevant without making the conjunctive alternative relevant. In contexts where it's not relevant, it will indeed not be excluded.

(66) a. Maximal sets of alternatives in $Alt((p \vee q) \,\Box\!\!\rightarrow r)$ that can be assigned *false* consistently with $(p \vee q) \,\Box\!\!\rightarrow r$:
(i) $\{p \,\Box\!\!\rightarrow r, (p \wedge q) \,\Box\!\!\rightarrow r\}$
(ii) $\{q \,\Box\!\!\rightarrow r, (p \wedge q) \,\Box\!\!\rightarrow r\}$
b. $IE((p \vee q) \,\Box\!\!\rightarrow r, Alt((p \vee q) \,\Box\!\!\rightarrow r))) = \bigcap$ (66a) $= \{(p \wedge q) \,\Box\!\!\rightarrow r\}$

Applying $\mathcal{E}xh^{\text{IE}}$ recursively would only give us the negation of the conjunctive alternative and nothing more.[43] But since the *Cell* interpretation is non-contradictory, by applying $\mathcal{E}xh^{\text{IE+II}}$ we get the falsity of the IE alternative and the truth of the non-IE ones. We thus derive SDA by Inclusion (see also left side of Fig. 4 below):

(67) $\mathcal{E}xh^{\text{IE+II}}_{Alt((p\vee q)\Box\!\rightarrow r)}(p \vee q) \,\Box\!\!\rightarrow r$
$\Leftrightarrow (p \,\Box\!\!\rightarrow r) \wedge (q \,\Box\!\!\rightarrow r) \wedge \neg((p \wedge q) \,\Box\!\!\rightarrow r)$

Deriving SDA by Innocent Inclusion further obviates several problems for previous implicature accounts of SDA. As Franke points out, his account fails to derive SDA directly when the antecedent contains more than two disjuncts (just as in FC; see Fox and Katzir 2018 for details):

(68) If you eat an apple, an orange, or a pear, you will be healthy. $(p \vee q \vee r) \,\Box\!\!\rightarrow s$
a. $\rightsquigarrow$ If you eat an apple you will be healthy. $p \,\Box\!\!\rightarrow s$
b. $\rightsquigarrow$ If you eat an orange you will be healthy. $q \,\Box\!\!\rightarrow s$
c. $\rightsquigarrow$ If you eat a pear you will be healthy. $r \,\Box\!\!\rightarrow s$

With Innocent Inclusion, on the other hand, none of the disjunctive alternatives is IE in this case, since the truth of the prejacent together with the falsity of any two disjunctive alternatives (e.g., $q \,\Box\!\!\rightarrow s$ and $r \,\Box\!\!\rightarrow s$) ensures the truth of the remaining disjunctive alternative (e.g., $p \,\Box\!\!\rightarrow s$). So none of them is IE, and the falsity of all the IE (conjunctive) alternatives is consistent with their truth, hence they are assigned *true*.

Another problem arises for Franke when a conditional with a disjunctive antecedent is embedded in the scope of a universal quantifier, as in (69). Put differently, the problem of universal FC is here replicated with universal SDA. Franke (2011) doesn't account for these inferences globally (and a local derivation is not available in his framework).[44]

[43] This is since the exhaustified disjunctive alternatives end up falsifying the prejacent (which, unlike in the case of FC, is not entailed by them); as a result their falsity on the second layer of exhaustification becomes vacuous. We illustrate this here for the disjunctive alternative $p \,\Box\!\!\rightarrow r$:

(i) a. $\mathcal{E}xh^{\text{IE}}_{Alt((p\vee q)\Box\!\rightarrow r)} p \,\Box\!\!\rightarrow r \Leftrightarrow (p \,\Box\!\!\rightarrow r) \wedge \neg(q \,\Box\!\!\rightarrow r) \wedge \neg((p \vee q) \,\Box\!\!\rightarrow r) \wedge \neg((p \wedge q) \,\Box\!\!\rightarrow r)$
b. $((p \vee q) \,\Box\!\!\rightarrow r) \wedge \neg \mathcal{E}xh^{\text{IE}}_{Alt((p\vee q)\Box\!\rightarrow r)} p \,\Box\!\!\rightarrow r \Leftrightarrow ((p \vee q) \,\Box\!\!\rightarrow r)$

[44] As Franke (2011, pp. 55–58) points out, he does not capture universal FC without stipulations; for precisely the same reasons he does not capture universal SDA. The inability of his system to derive universal FC/SDA persists even if we move from the set of alternatives he assumes (with no multiple replacements) to the one we do.

(69) Everyone will feel guilty if they eat ice cream or cake. $\forall x((Px \vee Qx) \;\Box\!\!\rightarrow Rx)$

a. $\rightsquigarrow$ Everyone will feel guilty if they eat ice cream. $\forall x(Px \;\Box\!\!\rightarrow Rx)$

b. $\rightsquigarrow$ Everyone will feel guilty if they eat cake. $\forall x(Qx \;\Box\!\!\rightarrow Rx)$

The universal SDA example in (69) is accounted for, however, along the same lines of our account of universal FC. Assuming that *every* generates *some*-alternatives, the universal disjunctive alternatives end up II, yielding the inferences in (69a) and (69b).

Finally, Klinedinst's (2007) approach derives SDA locally, namely by strengthening the antecedent itself. Santorio (2016) claims that this assumption is problematic when considering conditionals with disjunctive antecedents in DE contexts, as in (70). To capture the inferences of such sentences, this putative local implicature will have to be derived under negation since, given a variably strict analysis, (70a) and (70b) don't follow from the basic semantics (in this respect SDA is different from FC).

(70) It's not true that you will feel guilty if you eat ice cream or cake. $\neg((p \vee q) \;\Box\!\!\rightarrow r)$

a. $\rightsquigarrow$ It's not true that you will feel guilty if you eat ice cream. $\neg(p \;\Box\!\!\rightarrow r)$

b. $\rightsquigarrow$ It's not true that you will feel guilty if you eat cake. $\neg(q \;\Box\!\!\rightarrow r)$

While we are not convinced that this is a decisive argument against Klinedinst's account,[45] we want to point out that with Innocent Inclusion there is no issue to begin with, since our derivation is at the global level, above negation.[46] The derivation is similar to the positive case in (61), assuming that the only alternatives of

[45] As noted by Klinedinst, this local implicature would be derived in a non-monotonic context rather than a downward entailing one (given the non-monotonicity of conditionals within a variably strict semantics). Importantly, deriving implicatures in DE contexts normally leads to a weakening of the meaning at the global level; in the current case it does not. It is thus essentially different than computing an implicature in a DE context.

[46] Another argument by Santorio against implicature accounts, however, holds also for globalist accounts such as Franke (2011) and our own proposal. The argument comes from *probably*-conditionals:

(i) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

a. $\rightsquigarrow$ If the winning ticket is between 1 and 70, probably Sarah won.
b. $\rightsquigarrow$ If the winning ticket is between 31 and 100, probably Sarah won.

As Santorio argues, the sentence could be true if both simplifications are true but what is taken on our approach to be the prejacent is false. We suspect however that the problem is not unique to conditionals and thus the solution should not be hardwired into the semantics of conditionals as Santorio's is. The same effect can be seen with *most* (the focus of Sect. 8.3). Suppose there are 7 kids, such that 3 of them are both on team A and team B, 2 of them are only on team A, and the remaining two are only on team B, and there are no other people on either team. The situation can be described as in (ii).

(ii) Most kids on team A or team B are on both teams.

a. $\rightsquigarrow$ Most kids on team A are on both teams.
b. $\rightsquigarrow$ Most kids on team B are on both teams.

Here too, it seems that the sentence can be true in virtue of (iia) and (iib) being true, even though it is false that most of the 7 kids are on both teams (which is equivalent to (ii)). We thank Paolo Santorio for pointing out this problem and wish there was more we could say. We return to simplification with *most* in Sect. 8.3.

$\neg((p \vee q) \;\Box\!\!\rightarrow r)$ are the disjunctive and conjunctive ones.[47] Since $\neg((p \vee q) \;\Box\!\!\rightarrow r)$ entails the disjunction of $\neg(p \;\Box\!\!\rightarrow r)$ and $\neg(q \;\Box\!\!\rightarrow r)$, none of them is IE and the only IE alternative is the conjunctive one $\neg((p \wedge q) \;\Box\!\!\rightarrow r)$.[48] The *Cell* interpretation is non-contradictory, and therefore the disjunctive alternatives are assigned *true*. By providing global derivations for conjunctive meanings of disjunction we then not only account for universal FC, which cannot be derived globally by Klinedinst; we also avoid problems he might have with deriving SDA.

The idea that what's responsible for SDA is the disjunctive alternatives is not unique to implicature accounts of SDA; it is shared with alternative semantics–based approaches, most notably Alonso-Ovalle (2009). Our account (along with Klinedinst 2007; Franke 2011) differs from such approaches in not hard-wiring quantification over such alternatives into the system. Instead, SDA follows from the independently needed exhaustification mechanism we have argued for based on FC phenomena. Moreover, by not hard-wiring quantification over the disjunctive alternatives into the system, our approach makes distinct predictions, which we investigate in Sect. 8.

Our Inclusion-based account of simplification is highly dependent on the shape of the alternatives generated and on the entailment relations among them. In Sect. 8.1 we focus on predictions pertaining to the entailment relations and argue that they are verified by the absence of simplification inferences in an environment identified by McKay and van Inwagen (1977). We then move on in Sect. 8.2 to discuss predictions pertaining to the shape of the alternatives, focusing on a puzzle discovered by Ciardelli et al. (2018). We introduce a proposal due to Schulz (2018) and explain how it can be taken to verify our prediction. Finally, our account predicts simplification inferences to be found outside the domain of conditionals; Sect. 8.3 discusses simplification of *most* with a disjunctive restrictor as a relevant case.

8 Further predictions of simplification by Inclusion

8.1 Failure of simplification

While SDA seems to hold quite generally, there are certain cases where it doesn't.[49] It has been observed that sometimes conditionals with disjunctive antecedents are true even though one of their simplifications is false. This puzzle has been pointed out by McKay and van Inwagen (1977), based on the following example:

[47] In Sect. 8.2 we assume that negation triggers $\mathcal{E}xh$ as an alternative. As the reader may verify, adding such alternatives in the case at hand would change nothing.

[48] Here too the exclusion of the conjunctive alternative is not a necessary inference of (70).

[49] The bulk of the ideas in this section came up in a discussion with Itai Bassi, to whom we are greatly indebted.

(71) If Spain had fought with the Axis or with the Allies, it would have been with the Axis.

a. $\rightsquigarrow$ If Spain had fought with the Axis it would have been with the Axis. (trivially)

b. $\not\rightsquigarrow$ # If Spain had fought with the Allies it would have been with the Axis.

McKay and van Inwagen (1977) argue against the semantic validity of SDA based on the acceptability of (71).[50] Nute (1980) contrasts (71) with (72), for which SDA seems to be valid, thus leading to oddity given world knowledge that Hitler would have been pleased only if Spain had fought with the Axis, not with the Allies.[51]

(72) #If Spain had fought with the Axis or with the Allies, Hitler would have been pleased.

a. $\rightsquigarrow$ If Spain had fought with the Axis, Hitler would have been pleased.

b. $\rightsquigarrow$ # If Spain had fought with the Allies, Hitler would have been pleased.

Let us now show that our account predicts a failure of simplification for (71), solely based on the fact that (71) has the form $(p \vee q) \mathrel{\Box\!\!\rightarrow} p$, i.e., the consequent is equivalent to one of the disjuncts in the antecedent.[52] As we will immediately demonstrate, the attested difference between the non-SDA example (71) and the SDA example (72) is predicted on our account since the entailment relations between the alternatives change and, consequently, the result we get by applying $\mathcal{E}xh^{\text{IE+II}}$ does too.

As we have seen, in a sentence of the form $(p \vee q) \mathrel{\Box\!\!\rightarrow} r$, such as (72), none of the disjunctive alternatives is IE. This is because there are two maximal sets of alter-

[50] See Lassiter (2018) for a more elaborate argument against the semantic validity of SDA based on examples of the following sort:

(i) If Spain had fought with the Axis or the Allies, it's likely, but not certain, that it would have fought with the Axis.

In the following discussion we set aside such cases and the complications they give rise to in determining both the basic semantics and the alternatives generated.

[51] Nute (1980) further observed that the status of (72) improves if it appears as a continuation to (71), as in (i). Regrettably we have nothing to say about this effect.

(i) If Spain had fought with the Axis or with the Allies, it would have been with the Axis. So if Spain had fought with the Axis or with the Allies, Hitler would have been pleased.

[52] More generally, following Nute (1980), we assume that SDA fails whenever the conditional has the form $(p \vee q) \mathrel{\Box\!\!\rightarrow} p^+$ (where p^+ logically entails p). This generalization captures the acceptability of (71) as well as (i), also discussed by Nute, in which the consequent is strictly stronger than the first disjunct in the antecedent. We will not explicitly discuss such cases; the interested reader may verify that the explanation to be provided here for (71) extends to (i), the key fact being that $(p \vee q) \mathrel{\Box\!\!\rightarrow} p^+$ entails $p \mathrel{\Box\!\!\rightarrow} p^+$. See Bar-Lev (2018, Chap. 1) for complete derivations.

(i) If Spain had fought with the Axis or with the Allies, it would have been with the Axis and Hitler would have been pleased.

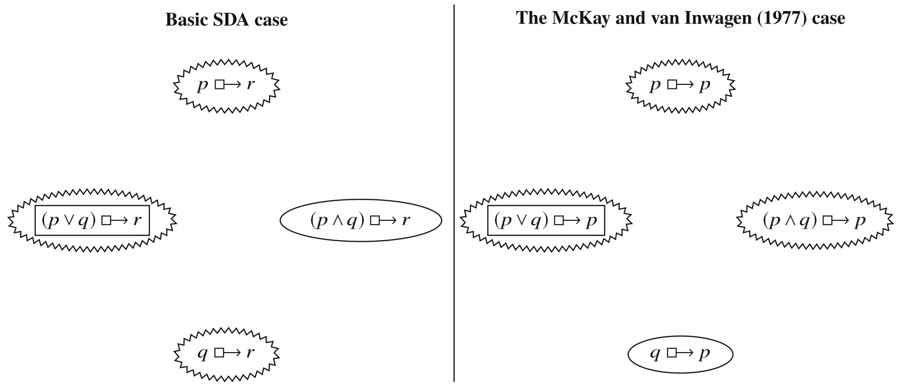


Fig. 4 Results of Innocent Exclusion and Innocent Inclusion for conditionals with disjunctive antecedents (notation as in Fig. 2). On the left: results when the consequent is logically independent from the antecedent, e.g., (72). On the right: results when the consequent is equivalent to one of the disjuncts in the antecedent, e.g., (71). [Indication of entailment relations suppressed.]

natives that can be assigned *false* consistently with the prejacent, as in (73) (repeated from (66a)):

(73) a. $\{p \mathrel{\Box\!\!\rightarrow} r, (p \wedge q) \mathrel{\Box\!\!\rightarrow} r\}$
b. $\{q \mathrel{\Box\!\!\rightarrow} r, (p \wedge q) \mathrel{\Box\!\!\rightarrow} r\}$

Since none of the disjunctive alternatives is in both sets, none of them is IE. Not being IE, they might be II, and as we have seen they indeed are.

Example (71), however, has a slightly different structure, that of $(p \vee q) \mathrel{\Box\!\!\rightarrow} p$. The alternatives we derive for such a structure would then be $p \mathrel{\Box\!\!\rightarrow} p$, which is a tautology, the contingent proposition $q \mathrel{\Box\!\!\rightarrow} p$, and the conjunctive alternative $(p \wedge q) \mathrel{\Box\!\!\rightarrow} p$, which is a tautology as well. There is therefore only one maximal set of alternatives that can be assigned *false* consistently with the prejacent:[53]

(74) $\{q \mathrel{\Box\!\!\rightarrow} p\}$

The proposition in this set would then be IE, and there would be nothing left for Inclusion to do. The results for the two cases are then as follows (see also Fig. 4):[54]

[53] With this line of reasoning one would expect to find other cases, unrelated to simplification, where the tautological nature of one of the alternatives leads to other alternatives becoming IE. However, we did not yet find such cases which are not predicted to be bad for independent reasons.

[54] The exclusion inference $\neg(q \mathrel{\Box\!\!\rightarrow} p)$ in (75b) is contextually redundant for (71), given world knowledge that fighting with one side (usually) entails not fighting with the rival side. It would, however, be detectable if it wasn't contextually entailed, as in (i) (uttered in a context where it is common ground that Mary studies physics).

(i) If Mary had studied linguistics or history, she would have studied linguistics.

(75) a. Result for conditionals of the form $(p \vee q) \;\Box\!\!\rightarrow r$, as in (72):
$\mathcal{E}xh^{\text{IE+II}}_{Alt((p\vee q)\Box\!\rightarrow r)}(p \vee q) \;\Box\!\!\rightarrow r \Leftrightarrow (p \;\Box\!\!\rightarrow r) \wedge (q \;\Box\!\!\rightarrow r) \wedge \neg((p \wedge q) \;\Box\!\!\rightarrow r)$ [=(67)]

b. Result for conditionals of the form $(p \vee q) \;\Box\!\!\rightarrow p$, as in (71):
$\mathcal{E}xh^{\text{IE+II}}_{Alt((p\vee q)\Box\!\rightarrow p)}(p \vee q) \;\Box\!\!\rightarrow p \Leftrightarrow ((p \vee q) \;\Box\!\!\rightarrow p) \wedge \neg(q \;\Box\!\!\rightarrow p)$

The solution to the puzzle—the contrast between McKay and van Inwagen's example in (71) in which SDA does not go through and the example in (72) in which it does—is due to the fact that only (71) generates a disjunctive alternative which is entailed by the prejacent. The existence of this alternative breaks the symmetry that usually holds between disjunctive alternatives and thereby derives a different result.

For the sake of completeness we should mention that even though Franke only mentions the McKay and van Inwagen (1977) case in passing, his 'iterated best response' paradigm makes the same predictions as ours as long as there are only two disjuncts in the antecedent, and shares the correct result that this case should behave differently than regular SDA cases.[55]

8.2 Turning switches

In a recent paper, Ciardelli et al. (2018) have discussed an intriguing difference in interpretation between the following two sentences (keeping with their notation, we write p, q for *A is up*, *B is up*, respectively, and $\bar{p}, \bar{q}$ for *A is down*, *B is down*, respectively).

(76) If switch A or switch B were down, the light would be off. $(\bar{p} \vee \bar{q}) \;\Box\!\!\rightarrow r$

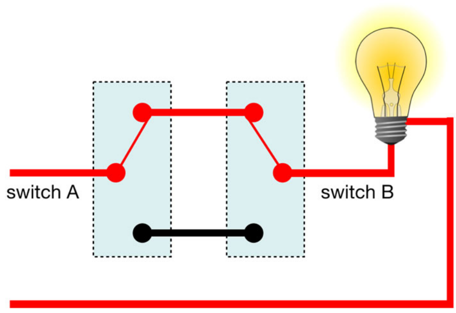
(77) If switch A and switch B were not both up, the light would be off. $(\neg(p \wedge q)) \;\Box\!\!\rightarrow r$

Footnote 54 continued
It is quite difficult to imagine (i) being uttered truthfully by a speaker who believes that if Mary had studied linguistics or history she would have definitely studied linguistics, but possibly also history. The exclusion inference predicted by applying $\mathcal{E}xh^{\text{IE+II}}$ (together with the assumption that $\neg(q \;\Box\!\!\rightarrow p) \Rightarrow q \;\Box\!\!\rightarrow \neg p$, which follows, for example, if Conditional Excluded Middle holds, as it does on Stalnaker's analysis) correctly precludes this possibility. This inference can be avoided though; Kai von Fintel (p.c.) pointed out that (iia) is felicitous. This behavior turns out to be predicted assuming that *linguistics or both* is parsed as *$\mathcal{E}$xh(linguistics) or both* (see Sect. 8.2). While the infelicity of (iib) may make the exclusion inference look like an obligatory one, we believe this is an independent issue having to do with felicity conditions on the use of *at least*, as is suggested by the felicity of (iic). We thank Benjamin Spector for providing (c) and this perspective on (iib).

(ii) a. If Mary had studied linguistics or history, she would have studied linguistics or both.
b. #If Mary had studied linguistics or history, she would have studied at least linguistics.
c. If Mary had studied linguistics or history or philosophy, she would have studied at least linguistics AND philosophy.

[55] This is not the case for Klinedinst (2007): for him the implicature leading to simplification is derived at an embedded level, within the antecedent, and subsequently cannot be affected by the identity of the consequent.

Fig. 5 Ciardelli et al.'s (2018) scenario. The picture is taken from their work and reproduced under CC BY 4.0



Ciardelli et al. discuss the situation depicted in Fig. 5. In this situation both switches are up right now. If only switch A was down or only switch B was down, the light would be off. But if both were down, the light would be on. They show experimentally that while (76) is mostly judged true in this scenario, (77) is mostly judged false or neither-true-nor-false. This difference is surprising since the antecedents in the two sentences are truth-conditionally equivalent (given that the switches cannot be neither up nor down). We wish to sharpen the puzzle by considering another example, which is minimally different from (76):

(78) If switch A or switch B or both were down, the light would be off.

According to our judgments, (78) is clearly false in Ciardelli et al.'s scenario. Moreover, this judgment seems clearer than the judgment for (77), which we waver on (a feeling consistent with Ciardelli et al.'s data, where (77) is not unanimously judged not true). Note, however, that the antecedents in (76), (77), and (78) are all semantically equivalent, and yet the truth judgments are clearly different. Most plausibly, the falsity of (78) is due to the simplification inference that *if both switches were down, the light would be off*. But how does this inference come about for (78), and why isn't it derived for (76)?

We expect to get parallel results for all of the conditionals in (76)–(78), given our view of simplification, only if we assume that they all have completely parallel sets of alternatives. But if the alternatives were different, we might get different results. Specifically, the conjunctive alternative equivalent to *If both switches were down, the light would be off* might cease to be IE given different alternatives, and possibly become II. In what follows we explore this route.

We will account for the facts as follows: First, we will claim that given independently needed assumptions, we expect the set of alternatives of (78) to be different than that of (76), and as a result the conjunctive alternative is II in this case. Second (following the spirit of Schulz 2018), we will employ the same machinery to explain Ciardelli et al.'s (77) and its intermediate status.

Chierchia et al. (2012) have argued that disjunctions of the form *P or Q or both*, as in the antecedent of (78), involve obligatory exhaustification of the first disjunct *P or Q* (due to Hurford's constraint against disjunctions in which one of the disjuncts entails the other). In other words, such disjunctions are obligatorily parsed as $\mathcal{E}xh(P$ *or* $Q)$ *or both*. Assuming this, (78) would have the form of $(\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \mathrel{\Box\!\!\rightarrow} r$. Note that this move does not make any difference for the truth conditions of the antecedent and hence the prejacent under our assumptions: the prejacent is still equivalent to $(\bar{p} \vee \bar{q}) \;\Box\!\!\rightarrow\; r$. It will, however, change the alternatives generated for (78), some of which would now have an embedded $\mathcal{E}xh$:[56]

(79) $$Alt((\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r) = \{\underline{(\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r}, (\mathcal{E}xh(\bar{p}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r, (\mathcal{E}xh(\bar{q}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r, \underline{(\bar{p} \wedge \bar{q}) \Box\!\!\rightarrow r}, \mathcal{E}xh(\bar{p} \vee \bar{q}) \Box\!\!\rightarrow r, \underline{\mathcal{E}xh(\bar{p}) \Box\!\!\rightarrow r}, \underline{\mathcal{E}xh(\bar{q}) \Box\!\!\rightarrow r}\}$$

To facilitate the presentation, we will only consider some of the alternatives generated and pretend that the set of alternatives is smaller, as in (80), which contains the underlined alternatives in (79) (simplified); the reader may verify that the result doesn't change with the full set of alternatives in (79).

(80) $$Alt((\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r) = \{(\bar{p} \vee \bar{q}) \Box\!\!\rightarrow r, (\bar{p} \wedge \neg\bar{q}) \Box\!\!\rightarrow r, (\bar{q} \wedge \neg\bar{p}) \Box\!\!\rightarrow r, (\bar{p} \wedge \bar{q}) \Box\!\!\rightarrow r\}$$

The effect of having exhaustified alternatives is that the conjunctive alternative $(\bar{p} \wedge \bar{q}) \Box\!\!\rightarrow r$ is no longer IE as in the basic case of (76). Namely, there is a maximal set of alternatives that can be assigned *false* consistently with the prejacent which doesn't contain the conjunctive alternative, from which it follows that this alternative is non-IE. This is since there are now alternatives symmetric to it: the falsity of $(\bar{p} \wedge \neg\bar{q}) \Box\!\!\rightarrow r$ and $(\bar{q} \wedge \neg\bar{p}) \Box\!\!\rightarrow r$ is consistent with the truth of the prejacent, and together they entail the truth of $(\bar{p} \wedge \bar{q}) \Box\!\!\rightarrow r$.[57] In fact, no alternative is IE:

(81) a. Maximal sets of alternatives in $Alt((\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r)$ that can be assigned *false* consistently with $(\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r$:

(i) $\{(\bar{p} \wedge \neg\bar{q}) \Box\!\!\rightarrow r, (\bar{p} \wedge \bar{q}) \Box\!\!\rightarrow r\}$

(ii) $\{(\bar{q} \wedge \neg\bar{p}) \Box\!\!\rightarrow r, (\bar{p} \wedge \bar{q}) \Box\!\!\rightarrow r\}$

(iii) $\{(\bar{p} \wedge \neg\bar{q}) \Box\!\!\rightarrow r, (\bar{q} \wedge \neg\bar{p}) \Box\!\!\rightarrow r\}$

b. $$IE((\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r, Alt((\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r) = \bigcap (81a) = \varnothing$$

The *Cell* interpretation is non-contradictory: all the alternatives are II. We get then (see also Fig. 6):

(82) $$\mathcal{E}xh^{\mathrm{IE+II}}_{Alt((\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r)}(\mathcal{E}xh(\bar{p} \vee \bar{q}) \vee (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r \Leftrightarrow (\bar{p} \wedge \neg\bar{q}) \Box\!\!\rightarrow r \wedge (\bar{q} \wedge \neg\bar{p}) \Box\!\!\rightarrow r \wedge (\bar{p} \wedge \bar{q}) \Box\!\!\rightarrow r$$

[56] We omit the alternative with the contradictory antecedent $(\mathcal{E}xh(\bar{p} \vee \bar{q}) \wedge (\bar{p} \wedge \bar{q})) \Box\!\!\rightarrow r$ since, being a non-contingent proposition, it will not affect the result: if it is taken to be a tautology as in Stalnaker (1968), for example, it will be trivially II. It is possible that more alternatives are generated, ones with no $\mathcal{E}xh$: $\bar{p} \Box\!\!\rightarrow r$ and $\bar{q} \Box\!\!\rightarrow r$. Since these are equivalent to other alternatives, they would change nothing.

[57] This can be understood more easily if we consider what the closest $\bar{p} \vee \bar{q}$ world might be. It can be either a $\bar{p} \wedge \neg\bar{q}$ world, or a $\bar{q} \wedge \neg\bar{p}$ world, or a $\bar{p} \wedge \bar{q}$ world. If we assign *false* to $(\bar{p} \wedge \neg\bar{q}) \Box\!\!\rightarrow r$ and $(\bar{q} \wedge \neg\bar{p}) \Box\!\!\rightarrow r$, then the closest $\bar{p} \vee \bar{q}$ world cannot be a $\bar{p} \wedge \neg\bar{q}$ world or a $\bar{q} \wedge \neg\bar{p}$ world (if the prejacent is going to be true); it must be a $\bar{p} \wedge \bar{q}$ world—that is, the alternative $(\bar{p} \wedge \bar{q}) \Box\!\!\rightarrow r$ must be true.

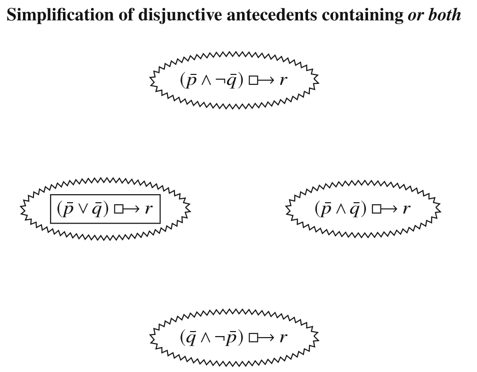


Fig. 6 Result of Innocent Exclusion and Innocent Inclusion for conditionals with a disjunctive antecedent containing *or both*, e.g., (78) (notation as in Fig. 2). [Indication of entailment relations suppressed.]

Importantly, we derive the truth of the alternative $(\bar{p} \wedge \bar{q}) \mathrel{\Box\!\!\rightarrow} r$, which explains the clear difference between (76) and (78). While the exhaustified meaning of the former entails the falsity of *If both switches were down the light would be off*, the exhaustified meaning of the latter entails its truth.[58] Since this conditional statement is indeed false in Ciardelli et al.'s scenario, the difference in judgments is predicted. The independently motivated assumption that (78) contains an embedded $\mathcal{E}xh$ has led to the derivation of the desired simplification inference $(\bar{p} \wedge \bar{q}) \mathrel{\Box\!\!\rightarrow} r$ for this case. We turn now to the case of (77), and propose a similar derivation to that of (78).

Schulz (2018) suggested to account for Ciardelli et al.'s contrast between (76) and (77) by blaming negation, which is present in (77) but not in (76), for generating more alternatives. Her solution is couched in a framework in which the alternatives generated by the antecedent are quantified over by the conditional operator (see, e.g., Alonso-Ovalle 2009; Santorio 2016). However, Schulz essentially stipulates that negated conjunction triggers more alternatives than disjunction, and it is not clear how this result could be achieved in a more principled fashion within her framework. We thus propose to convert Schulz's idea into the current framework, and implement it in the following way: we assume that negation triggers $\mathcal{E}xh$ as an alternative.[59] This assumption yields a very similar result to what we have just derived for (78), since the set of alternatives will be (almost) the same. The alternatives we get for (77) under this assumption (and the assumption that $Alt(p \wedge q) = Alt(p \vee q)$, which we adopted for the derivation of negative universal FC at the end of Sect. 5.4) are as follows:

[58] The ultimate story is a bit more nuanced. As we know from fn. 42, the falsity of the conjunctive alternative is derived for (76) only if it's relevant. The truth of the conjunctive alternative in the case of (78), by contrast, is an obligatory inference; see Sect. 9.1.

[59] Of course, this assumption should follow from a general theory of alternative generation such as Katzir (2007). A more in-depth discussion of this issue is something that we hope to return to in the future. Another issue is that so-called 'indirect implicatures', e.g., from *Not all students came* to *Some students came*, are not derivable if alternatives where negation is replaced with $\mathcal{E}xh$ are generated. As we mention towards the end of this section, we assume that whether such alternatives are generated depends on whether negation is contained in a focus-marked constituent or not; we'd hence expect indirect implicatures only if it's not. See Chierchia (2004) and Romoli (2012) for conflicting opinions as to the status of indirect implicatures relative to standard ones.

(83) $Alt((\neg(p \wedge q)) \,\Box\!\!\rightarrow r) =$
$\{(\neg(p \wedge q)) \,\Box\!\!\rightarrow r, (\neg p) \,\Box\!\!\rightarrow r, (\neg q) \,\Box\!\!\rightarrow r, (\neg(p \vee q)) \,\Box\!\!\rightarrow r,$
$(\mathcal{E}xh(p \vee q)) \,\Box\!\!\rightarrow r, (\mathcal{E}xh(q)) \,\Box\!\!\rightarrow r, (\mathcal{E}xh(p)) \,\Box\!\!\rightarrow r, (\mathcal{E}xh(p \wedge q)) \,\Box\!\!\rightarrow r\}$

As the reader may verify (bearing in mind that $\neg p = \bar{p}$ and $\neg q = \bar{q}$, i.e., *not up=down*), the alternatives in (83) and (79) yield the same propositions, except for the last alternative in (83), which has no parallel in (79). For essentially the same reasons as in (78) then, the disjunctive alternative $(\neg(p \vee q)) \,\Box\!\!\rightarrow r$ is non-IE: the alternatives $(\mathcal{E}xh(q)) \,\Box\!\!\rightarrow r$ and $(\mathcal{E}xh(p)) \,\Box\!\!\rightarrow r$ are symmetric to it. As it turns out (modulo the last alternative in (83), which ends up IE but changes nothing otherwise), the two cases are the same.

(84) $\mathcal{E}xh^{\text{IE+II}}_{Alt((\neg(p \wedge q)) \Box\!\rightarrow r)}(\neg(p \wedge q)) \,\Box\!\!\rightarrow r \Leftrightarrow$
$(p \wedge \neg q) \,\Box\!\!\rightarrow r \wedge (q \wedge \neg p) \,\Box\!\!\rightarrow r \wedge (\neg(p \vee q)) \,\Box\!\!\rightarrow r \wedge \neg((p \wedge q) \,\Box\!\!\rightarrow r)$

Importantly, we derive the truth of the alternative $(\neg(p \vee q)) \,\Box\!\!\rightarrow r$, which explains the significant decrease in true judgments for (77) relative to (76). Recall, however, that while (77) was mostly judged false or neither-true-nor-false, its non-truth was not as clear as that of (78). We submit that this is due to the possible effects of focus on the alternatives (Rooth 1992).[60] Specifically, we would like to adopt the proposal in Fox and Katzir (2011) and Katzir (2014) according to which alternatives are derived by substitution within focus-marked constituents. Now if negation is within a focus-marked constituent, then it will be replaced with $\mathcal{E}xh$, the result we derived above will hold, and the sentence will be false. If negation is not within a focus-marked constituent, then no replacement of negation with $\mathcal{E}xh$ will be triggered, the set of alternatives for $(\neg(p \wedge q)) \,\Box\!\!\rightarrow r$ will be identical to that of $(\bar{p} \vee \bar{q}) \,\Box\!\!\rightarrow r$, and the sentence will be true.

8.3 Simplification with *most*

Our Inclusion-based account of SDA isn't tied to the semantics of conditionals and hence predicts simplification inferences to surface outside this domain. Our goal in this section is to investigate this prediction for a construction in which *most* has a disjunctive restrictor. As we will demonstrate below, we predict (85) to give rise to the inferences in (85a) and (85b).[61]

[60] Another reason might be that unlike in the case of (78), where the conjunctive alternative can be arrived at by merely deleting parts of the prejacent, its parallel in the case of (77) cannot. See Sect. 9.1 for the possible relevance of this difference.

[61] Another potentially relevant case of a simplification problem is with superlatives:

(i) I climbed the highest mountains in North America or South America.

We have not yet fully investigated the applicability of Innocent Inclusion to such cases.

(85) Most students in linguistics or philosophy took Advanced Syntax. $Most(P \cup Q)(R)$

a. $\rightsquigarrow$ Most students in linguistics took Advanced Syntax. $Most(P)(R)$

b. $\rightsquigarrow$ Most students in philosophy took Advanced Syntax. $Most(Q)(R)$

According to our intuitions, these are indeed inferences of (85). Consider the following scenario: 35 out of the 40 linguistics students and 2 out of the 30 philosophy students took Advanced Syntax (and there are no linguistics-and-philosophy students). In this case (85) seems odd (or false) even though more than half of the members in the union of linguistics students and philosophy students took Advanced Syntax (37 out of 70; compare with: *Most students in the department took Advanced Syntax*). This, presumably, is since the inference in (85b) is false.[62,63] Admittedly, the intuition that the sentence is odd in the above scenario is not shared by all speakers, in contrast with SDA. While further investigation into the status of such inferences is needed, their existence requires an explanation—which we will now show is provided by Innocent Inclusion.

For expository purposes we assume the following semantics for *most*, the crucial ingredient being that it is non-monotonic with respect to its restrictor:

(86) $$Most(P)(Q) = 1 \text{ } \mathit{iff} \text{ } \frac{|P \cap Q|}{|P|} > \frac{1}{2}$$

The set of alternatives we assume for $most(P \cup Q)(R)$ is in (87), considering not only the disjunctive and conjunctive alternatives but also alternatives where *most* is replaced with *some*, similarly to our discussion of universal FC above.[64] (Without the *some*-alternatives $most(P)(R)$ and $most(Q)(R)$ would end up IE rather than II.[65])

(87) $$Alt(most(P \cup Q)(R)) = \{most(P \cup Q)(R), most(P)(R), most(Q)(R), most(P \cap Q)(R), some(P \cup Q)(R), some(P)(R), some(Q)(R), some(P \cap Q)(R)\}$$

The IE alternatives are as follows:

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62 For reasons we do not understand, embedding disjunction in a relative clause seems to make the sentence less odd in this scenario:

(i) Most students who study linguistics or philosophy took Advanced Syntax.

63 Here too we can generate a McKay and van Inwagen-style effect, as seen in (i). As with simplification in conditionals, we infer that it is false that *most philosophy students are linguistics students* (see fn. 54). For elaboration on this see Bar-Lev (2018, Chap. 1).

(i) Most students in linguistics or philosophy are linguistics students. $Most(P \cup Q)(P)$

64 *Most* would also trigger *all*-alternatives, of course, which we ignore since they only add more IE alternatives but have no effect on what's II, which is the focus of our discussion.

65 Since $most(P \cup Q)(R)$, $\neg most(P)(R)$, and $\neg most(Q)(R)$ can all be true, for instance if $P = \{a, c\}$, $Q = \{b, c\}$, and $R = \{a, b\}$.

(88) a. Maximal sets of alternatives in $Alt(most(P \cup Q)(R))$ that can be assigned *false* consistently with $most(P \cup Q)(R)$:
(i) $\{most(P)(R), most(Q)(R), most(P \cap Q)(R), some(P \cap Q)(R)\}$
(ii) $\{most(P)(R), some(P)(R), most(P \cap Q)(R), some(P \cap Q)(R)\}$
(iii) $\{most(Q)(R), some(Q)(R), most(P \cap Q)(R), some(P \cap Q)(R)\}$
b. $IE(most(P \cup Q)(R), Alt(most(P \cup Q)(R))) = \bigcap$(88a) $=$
$\{most(P \cap Q)(R), some(P \cap Q)(R)\}$

The *Cell* interpretation is non-contradictory, and exhaustification thus assigns *true* to the disjunctive alternatives. We thus capture the simplification inferences of (85).

(89) $\mathcal{E}xh^{\text{IE+II}}_{Alt(most(P \cup Q)(R))} most(P \cup Q)(R) \Leftrightarrow$
$most(P \cup Q)(R) \wedge most(P)(R) \wedge most(Q)(R) \wedge \neg some(P \cap Q)(R)$

8.4 Interim summary

We have considered several phenomena where the *Cell* interpretation is empirically correct yet not derived by previous accounts. We have argued on this basis in favor of our proposed mechanism, which combines Innocent Exclusion and Innocent Inclusion and delivers the *Cell* interpretation whenever it is non-contradictory. Before concluding, we discuss some remaining issues in Sect. 9: differences between inferences we lump together as resulting from exhaustification, and some problematic cases of embedded FC disjunction.

9 Some loose ends

9.1 Obligatoriness

Experimental work has revealed significant differences between FC inferences and run-of-the-mill scalar implicatures (Chemla 2009b; Chemla and Bott 2014). Similarly, SDA inferences seem more robust than run-of-the-mill scalar implicatures. To explain these differences, which prima facie conflict with implicature accounts of FC and SDA, Bar-Lev and Fox (2017) and Bar-Lev (2018, Chap. 2) hypothesized that FC and SDA are obligatory implicatures, while run-of-the-mill scalar implicatures are optional and subject to considerations of relevance. (For arguments for the existence of obligatory scalar implicatures, see Chierchia 2004 and much subsequent work.) We can't provide here a satisfactory account of obligatoriness. We would like, however, to mention two possible directions that could be pursued. One direction (explored in Bar-Lev and Fox 2017) is that Inclusion inferences are obligatory, whereas Exclusion ones are not. Another direction (pursued in Bar-Lev 2018, Chap. 2, following the lead of Chemla and Bott 2014) is that the distinction between obligatory and optional inferences tracks the distinction between inferences resulting from alternatives generated by deletion (i.e., replacing a constituent with a sub-constituent) and ones resulting from alternatives generated by substitution (i.e., replacing a constituent with an item from the lexicon;

see Katzir 2007; Fox and Katzir 2011). This distinction has been independently argued to be relevant for explaining children's intricate behavior with scalar implicatures (e.g., by Zhou et al. 2013; Tieu et al. 2016; Singh et al. 2016). See also Chemla and Singh (2014) for relevant discussion.

9.2 Other cases of embedded FC disjunction

In Sect. 5 we discussed universal FC in light of two arguments for a global account—one based on a parse in which $\mathcal{E}xh$ takes matrix scope. We pointed out, however, that the arguments only show that a global derivation is available, not that a local derivation—where $\mathcal{E}xh$ is embedded below a quantifier—is prohibited (Sect. 5.4). In fact, within the framework we are assuming, a special constraint would need to be invoked to block a local derivation, and no such constraint has ever been proposed (as far as we know). In this section, we point out that a local derivation is permitted and indeed needed not only on conceptual grounds, but also on empirical grounds: the global derivation we have argued for does not generalize to similar constructions where analogous inferences are attested.

In other words, the argument is based on the observation that the case of universal FC is but one manifestation of a more general pattern found whenever an FC construction is embedded in the scope of a quantifier. Quite generally, the following inference pattern appears to hold for any quantifier Q:[66]

(90) $Qx\Diamond(Px \vee Rx) \rightsquigarrow Qx\Diamond Px \wedge Qx\Diamond Rx$

We have seen that $\mathcal{E}xh^{\text{IE+II}}$ provides a global derivation for universal FC, namely it explains the inference pattern in (90) in cases where the Q at hand is universal. However, it does not provide a global derivation in many other cases, e.g., the 'existential FC' case in (91).[67]

(91) Some boys are allowed to eat ice cream or cake. $\exists x\Diamond(Px \vee Qx)$

a. $\rightsquigarrow$ Some boys are both allowed to eat ice cream and allowed to eat cake. $\exists x(\Diamond Px \wedge \Diamond Qx)$

The problem with a global derivation in this case is that it derives weaker inferences than those attested sometimes.[68] Rather than deriving that there are boys allowed ice cream and allowed cake (boys that have FC), we get the inferences that *some boys are allowed ice cream* and *some boys are allowed cake*, which can be true even if (91a) is false, namely if no boy has free choice.[69] So we would like to take these cases to

66 We thank an anonymous reviewer for suggesting this way of presentation.

67 Similar examples, with inference patterns similar to those discussed in this section, can be constructed using conditionals with disjunctive antecedents instead of FC disjunction.

68 We thank an anonymous SALT 27 reviewer for pointing out that FC disjunction under *some* still requires a local derivation.

69 No parallel issue for a global derivation arises with singular indefinites like *some boy*: in this case, the *only one* implicature associated with singular indefinites (namely that no more than one boy is allowed ice cream or cake; see fn. 14), together with the inference that some boy is allowed ice cream and some boy is

argue for the obvious conclusion that a local application of $\mathcal{E}xh$ is indeed available (i.e., in the scope of *some boys*).[70]

This leads to a very interesting prediction, which we have not been able to study as productively as we would have liked to. Specifically, we predict that if a local derivation is blocked, by the two methods discussed in Sect. 5 (in the context of the arguments that a global derivation is available as well), we will get different results. In other words, we expect to only get in such cases the weaker inference $\exists x \Diamond Px \wedge \exists x \Diamond Qx$, rather than the stronger one attested in (91a).

One method to block a local derivation (from Chemla 2009b, see Sect. 5.2 above) invoked negative counterparts of the relevant sentences where a local position for $\mathcal{E}xh$ was not available. Using that method, we expect that a negative counterpart of a sentence of the form $\exists x \Diamond (Px \vee Qx)$, as in (92), would support the inferences in (92a) and (92b), but not the inference in (92c) (which is parallel to (91a)).

(92) Not every student is required to solve both problem A and problem B. $\neg \forall x \Box (Px \wedge Qx) \Leftrightarrow \exists x \Diamond (\neg Px \vee \neg Qx)$

a. $\rightsquigarrow$ Not every student is required to solve problem A. $\neg \forall x \Box Px \Leftrightarrow \exists x \Diamond \neg Px$

b. $\rightsquigarrow$ Not every student is required to solve problem B. $\neg \forall x \Box Qx \Leftrightarrow \exists x \Diamond \neg Qx$

c. $\not\rightsquigarrow$ Some student is allowed to avoid solving problem A and allowed to avoid solving problem B. $\exists x (\Diamond \neg Px \wedge \Diamond \neg Qx)$

While the judgments are difficult, given the complexity of the sentences involved, our judgments are in line with this expectation.

The second method used in order to argue for the availability of a global derivation was based on VP-ellipsis considerations (in Sect. 5.3). At first glance, this method yields the opposite conclusion from the one we think is supported by Chemla's method: it is possible, we think, to have an FC meaning under *some girls* while not having it on the elided part in the scope of *no boys*.

(93) Some girls are allowed to eat ice cream or cake on their birthday. Interestingly, no boys are ~~allowed to eat ice cream or cake on their birthday~~. $\approx$

a. Some girls are both allowed to eat ice cream *and* allowed to eat cake on their birthday, and

b. no boys are allowed to eat ice cream and (likewise) no boys are allowed to eat cake on their birthday.

This seems to be a problem for our proposal. However, we can think of two possible ways out. First, we could accept a global derivation for existential FC. As mentioned in fn. 70, if the indefinite is analyzed as a referential expression, a global derivation is

Footnote 69 continued
allowed cake, entails the desired inference that there is one and only one boy who's both allowed ice cream and allowed cake.

[70] An alternative way to address this problem would appeal to a particular theory of wide scope indefinites ('referential indefinites'; Kasher and Gabbay 1976, Fodor and Sag 1982, and much subsequent work). If *some boys* here is referential, the problem doesn't arise.

readily available. Second, we could argue that our considerations from Sect. 5.3 lead to a more nuanced picture in cases that involve quantification over plural domains.

In Sect. 5.3, we relied on the assumption that the binder of any elided bound variable has to be inside the parallelism domain for ellipsis. We could account for the facts in (93) if the binder is, in fact, not *some girls* but rather a distributivity operator applying in its scope. In this case $\mathcal{E}xh$ could outscope the distributivity operator, thus ending up outside the parallelism domain. Assuming that the distributivity operator is universal as standard, the prejacent taken by $\mathcal{E}xh$ would have the structure $\forall x \Diamond (Px \vee Qx)$. This is essentially a case of universal FC, which can be derived using $\mathcal{E}xh^{\text{IE+II}}$.[71]

Of course, we will have to examine how our two methods for forcing a global derivation apply to other quantificational phrases, assuming that the pattern in (90) holds more generally. We unfortunately have to leave this important issue for some other occasion. For a partial discussion of various quantificational environments and of what happens when a local occurrence of $\mathcal{E}xh$ is embedded under yet another occurrence of $\mathcal{E}xh$, see Bar-Lev (2018, Chap. 1.5).

10 Summary

We presented a novel theory of exhaustification, in which the goal of $\mathcal{E}xh$ is to deliver (whenever possible) a cell in the partition induced by the set of alternatives. To let $\mathcal{E}xh$ achieve this goal, we suggested that it must make use not only of Innocent Exclusion (Fox 2007) but of Innocent Inclusion as well. We have argued for Innocent Inclusion by considering a range of data that show that whenever Innocently Includable alternatives exist, their truth is inferred.

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[71] Note, however, that this would require assuming the existence of an existential alternative to the universal distributivity operator, raising questions similar in spirit to those that arose in our discussion of Nouwen (2017) and ability modals (Sect. 6.2).

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