NEW TRENDS IN FORMAL SEMANTICS

16TH FEBRUARY 2021

Free Choice and Homogeneity – Simon Goldstein (2019)

INTRODUCTION

- We are all aware of the phaenomenon of Free Choice:
  - You can take the apple or the pear
  - You can take the apple.

NARROW	**WIDE**
You can take the apple or the pear.	You can take the apple or you can take the pear.
$\Diamond (A \lor B)$	$\Diamond A \lor \Diamond B$
$\Diamond A \land \Diamond B$	$\Diamond A \land \Diamond B$

NOTE: THIS DISTINCTION WILL BE IMPORANT LATER

- Together with other apparently unproblematic principles, free choice leads to **explosion:**

FREE CHOICE		
$\Diamond (A \lor B) \vDash \Diamond A \land \Diamond B$		
DISJUNCTION INTRODUCTION	**DOUBLE PROHIBITION**	
$A \vDash A \lor B$	$\neg \Diamond (A \lor B) \vDash \neg \Diamond A \land \neg \Diamond B$	**EXPLOSION**
UPWARDS MONOTONICITY	**CONTRAPOSITION**	$\Diamond A \vDash \Diamond B$
If $A \vDash B$ then $\Diamond A \vDash \Diamond B$	If $A \vDash B$ then $\neg A \vDash \neg B$	
TRANSITIVITY		
If $A \vDash B$ and B $\vDash C$ then A $\vDash C$		

- There are different ways to deal with this problem:
    - We accept the principles above, but we treat *free choice* as a **pragmatic inference** (e.g. Alonso-Ovalle 2006) (or similarly, we treat *free choice* as a valid principle, and *double prohibition* as pragmatic)
    - We go for a **non-classical** treatment of some logical operators (e.g. Aloni 2018)
    - We add a hybrid semantic-pragmatic principle of **exhaustification** and we work with different set of alternatives (e.g. Fox et al. 2007, 2018, 2020)
- Goldstein tries to develop a new semantics:
    - All principles, **except for transitivity** are valid
    - **Homogeneity** effects (leads to undefined values)
    - **Strawson entailment** (it works with defined/undefined values)

HOMOGENEITY

- Homogeneity effects are typically observed in the case of **plurals**:

(3) John washed the dishes.

→ John washed all of them.

(4) John did not wash the dishes.

→ John washed none of them.

- We say that the group of dishes is *homogeneous* w.r.t. the property of having been washed by John.
- Observe also the contrasts with all-phrases

(5) John washed all the dishes.

(6) John did not wash all the dishes.

- Note that (6) is true when (5) is false. This does not apply to (3) and (4) and when John washed some of the dishes, (3) and (4) are neither true or false, but they are considered ***undefined***.
- The idea is that something similar happens with **free choice**:

(7) John can take the apple or the banana.

→ John can take either.

(8) John cannot take the apple or the banana.

→ John can take neither.

- How does Goldstein implement homogeneity effects in his analyses?
  - Homogeneus Alternative Semantics: **possibility modals**
  - Homogeneus Dynamic Semantics: **disjunction**

THE FRAMEWORKS

Alternative Semantics	Homogeneus Alternative Semantics
Meaning of a sentence = set of alternatives To validate Free Choice, possibility modals operate on each of the alternatives.	Meaning = set of total functions from worlds to {0, #, 1}, where # is the undefined value. To validate Free Choice, we assume that when $\Diamond\,(A \vee B)$ is true and *defined*, all the alternatives are true.
i. $[\![\mathsf{p}]\!] = \{\{w \mid w(\mathsf{p}) = 1\}\}$ ii. $[\![\neg\mathsf{A}]\!] = \{W - \bigcup[\![\mathsf{A}]\!]\}$ iii. $[\![\mathsf{A} \wedge \mathsf{B}]\!] = \{\mathbf{A} \cap \mathbf{B} \mid \mathbf{A} \in [\![\mathsf{A}]\!], \mathbf{B} \in [\![\mathsf{B}]\!]\}$ iv. $[\![\mathsf{A} \vee \mathsf{B}]\!] = [\![\mathsf{A}]\!] \cup [\![\mathsf{B}]\!]$ v. $[\![\Diamond\mathsf{A}]\!] = \{\bigcap\{\Diamond\mathbf{A} \mid \mathbf{A} \in [\![\mathsf{A}]\!]\}\}$	i. $[\![\mathsf{p}]\!] = \{\lambda w \,.\, w(\mathsf{p}) = 1\}$ ii. $[\![\neg\mathsf{A}]\!] = \{W - \bigcup[\![\mathsf{A}]\!]\}$ iii. $[\![\mathsf{A} \wedge \mathsf{B}]\!] = \{\mathbf{A} \cap \mathbf{B} \mid \mathbf{A} \in [\![\mathsf{A}]\!], \mathbf{B} \in [\![\mathsf{B}]\!]\}$ iv. $[\![\mathsf{A} \vee \mathsf{B}]\!] = [\![\mathsf{A}]\!] \cup [\![\mathsf{B}]\!]$ v. $[\![\Diamond\mathsf{A}]\!] = \{\lambda w : \exists v \in \{0,1\}\, \forall\mathbf{A} \in [\![\mathsf{A}]\!]\ \Diamond\mathbf{A}(w) = v \,.\, \forall\mathbf{A} \in [\![\mathsf{A}]\!]\ \Diamond\mathbf{A}(w) = 1\}$
$\mathsf{A}_1, \ldots, \mathsf{A}_n \models \mathsf{C}$ iff $\bigcap_{i \in [1,n]}(\bigcup[\![\mathsf{A}_i]\!]) \subseteq \bigcup[\![\mathsf{C}]\!]$	**Definition 7.** $\mathsf{A}_1, \ldots, \mathsf{A}_n \models \mathsf{C}$ iff $\bigcup[\![\mathsf{C}]\!](w) = 1$ if: i. $\forall i \in [1,n]\ \bigcup[\![\mathsf{A}_i]\!](w) = 1$ ii. $\bigcup[\![\mathsf{C}]\!](w) \in \{0,1\}$

Homogeneus Dynamic Semantics

- An information state s (set of possible worlds)
- A context change potential (partial function from s to a new information state)
- An interpretation function which maps every sentence to a context change potential

To validate Free Choice, we assume that s[A V B] is defined, only if either s support both possibilities or none of them.

Definition 11 (Homogeneous dynamic semantics).

i. $s[\mathrm{p}] = s \cap \{w \mid w(\mathrm{p}) = 1\}$

ii. $s[\neg \mathrm{A}] = s - s[\mathrm{A}]$

iii. $s[\mathrm{A} \wedge \mathrm{B}] = s[\mathrm{A}] \cap s[\mathrm{B}]$

iv. $s[\Diamond \mathrm{A}] = s \cap \{w \mid s[\mathrm{A}] \neq \emptyset\} \cup s[\mathrm{A}]$

v. $s[\mathrm{A} \vee \mathrm{B}] = \begin{cases} s[\mathrm{A}] \cup s[\mathrm{B}] & \text{if } s \models \Diamond \mathrm{A} \wedge \Diamond \mathrm{B} \text{ or } s \models \neg \Diamond \mathrm{A} \wedge \neg \Diamond \mathrm{B} \\ \# & \text{otherwise} \end{cases}$

Definition 12. $\mathrm{A}_1, \ldots \mathrm{A}_n$ entail C ($\mathrm{A}_1, \ldots \mathrm{A}_n \models \mathrm{C}$) just in case $s \models \mathrm{C}$ whenever:

i. $s \models \mathrm{A}_1; \ldots; s \models \mathrm{A}_n$.

ii. $s[\mathrm{C}] \neq \#$.

- An example: ***Double Prohibition***

$\neg \Diamond (A \vee B) \vDash \neg \Diamond A \wedge \neg \Diamond B$		
Alternative Semantics	Homogeneus Alternative Semantics	Homogeneus Dynamic Semantics
The principle is invalid: $\Diamond A$ is true, but $\Diamond B$ false	Suppose $\neg \Diamond (A \vee B)$ is true, then $\Diamond (A \vee B)$ is false. By homogeneity, all alternatives in $[\![A \vee B]\!]$ must be impossible.	Suppose s supports $\neg \Diamond (A \vee B)$. Then by definiteness and support, both $\Diamond A$ and $\Diamond B$ are not supported by s.

- His predictions:

		AS	HAS	HDS
Free Choice	$\Diamond(A \vee B) \models \Diamond A \wedge \Diamond B$	+	+	+
Double Prohibition	$\neg\Diamond(A \vee B) \models \neg\Diamond A \wedge \neg\Diamond B$	-	+	+
Wide Free Choice	$\Diamond A \vee \Diamond B \models \Diamond A \wedge \Diamond B$	-	-	+
Disjunction Introduction	$A \models A \vee B$	+	+	+
LEM	$\models A \vee \neg A$	+	+	+
IAT	$\models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$	-	-	-
Modal Disjunction	$A \vee B \models \Diamond A \wedge \Diamond B$	-	-	+
UM	$A \models B \Longrightarrow \Diamond A \models \Diamond B$	-	+	+
Transitivity	$A \models B \;\&\; B \models C \Longrightarrow A \models C$	+	-	-
Contraposition	$A \models B \Longrightarrow \neg B \models \neg A$	+	+	+

- So far, we have only examined the case of epistemic modality. In section 8, Goldstein extends his framework to **other modalities** by introducing a general modal operator ⧫ parametrised to different accessibility relations.
- In the case of **deontic modals** (e.g. may), we obtain different predictions in the case of wide scope free choice (section 9.1)

  (26) You may take an apple or you may take a pear, but I don't know which.
- Homogeneity and **quantifiers** (section 9.2):

  (30) Every philosopher or linguist went to the party

  (30) clashes with the homogeneity aspect of his account. (dynamic account of Px V Lx)
- Comparison with **other accounts** (section 10):
  - Aher 2012 and Aloni 2018
    - Some of them validate different set of formulas (e.g. Aher), while the analysis of Aloni validates all (?) the formulas valid in HDS.
    - In the case of Aloni, NE leads to issues with explosion (but note that there are different notions of contradictions in Aloni's account)
    - Two set of alternatives (positive and negative) as opposed to single set of trivalent alternatives. The role of Free Choice in negated conjunctions ($\neg(\neg \Diamond A \wedge \neg \Diamond B)$)
  - Implicature based accounts (Chemla 2008, Fox 2007):
    - The dual $\neg \Box (A \wedge B) \vDash \neg \Box A \wedge \neg \Box B$ has the same status of ordinary free choice
    - Wide Free choice $\Diamond A \vee \Diamond B$ fails (in the case of Fox)
    - Double Prohibition: based on Fox account, only one of the "prohibitions" must hold
    - Compossibility:

      (43) Jane may sing or dance.

- Discussion:
  - Is Free Choice really a homogeneity effect?

(17) Context: *Half of the professors smiled.*
A: The professors smiled.
B: Well / #yes / #no, half of them.

We can observe a parallel effect in the case of Free Choice.

(18) Context: *Soup is permitted; salad is not.*
A: Mary may have soup and she may have salad.
B: Well / #yes / no, she can have soup (but she can't have salad).

(19) Context: *Soup is permitted; salad is not.*
A: Mary may have soup or salad.
B: Well / #yes / #no, she can have soup (but she can't have salad).

Other interesting examples (relationship with the LEM):

(i) Adam either read the books or he didn't read them.

(ii) Well, what if he read half of the books?

Interestingly, the analogous argument in our case is unsuccessful:

(iii) Either you can have soup or salad, or you can't.

(iv) ?Well, what if I can have soup but I can't have salad?

  - Are we happy with dropping transitivity of entailment?
  - Up to what extent is this paper in line with the experimental literature?
  - Free choice is a multifaceted phenomenon: e.g. ability disjunction (I can write or read) and homogeneity.
  - Is it possible to implement homogeneity in a different way?
  - The paper discusses several proofs and technicalities.

POST-DISCUSSION NOTES

- **Narrow scope free choice and slucing.**

  Deontic wide scope free choice can be cancelled by adding a continuation like the one in (P1):

  (P1) You may take the apple or you may take the pear, but I do not know which one.

  Goldstein's account can deal with this (by assuming that the modal is parametrized with a non-universal accessibility relationship). (see p. 27)
  However, for **cancellability under narrow scope free choice**, there is no analogue way to capture the narrow scope equivalent:

  (P2) You may take the apple or the pear, but I do not know which one.

  For (P2), we must assume that at the level of the logical form, disjunction takes wide scope to the modal operator. The underlying reason of why this is the case is unaddressed in the paper.

  Regarding the issue of cancelling or suspending the FC inference, we have noticed that there are contextual ways to strengthen (P3) or weaken (P4) the FC inference. This was particularly relevant to **Terence's thesis topic**.

  (P3) You may take the apple or you may take the pear. Anything goes!

  (P4) You may take the apple or you may take the pear. But be careful with your choice!

- **Quantification**

  In Section 9.2, Goldstein acknowledges that his DHS is problematic when it comes to the interaction between **quantifiers and epistemic modality**. To solve this issue, Goldstein needs to redefine the epistemic modality operator in a way in which it is sensitive to a different update procedure. We were wondering if it is possible to deal with issue by working at the level of the quantifiers and the domain of quantification, rather than at the level of the single update operators. This issue is particularly relevant for the first-order implementation of Maria's account of Free Choice, a topic which **Maria and Peter** are currently working on.

- **Maria's take**

  In her works, Maria defended different accounts of free choice. She noted that the principles valid in HDS and her last account are the same, and we wondered if there is a relationship between a notion of homogeneity for free choice and her analysis. In this regard, she noted that there is strong underlying assumption behind Goldstein's account: **free choice is a homogeneity effect**. The data in this support might not be decisive and even the exact contribution of homogeneity at the theoretical level is debatable. Moreover, dropping **transitivity** of entailment is not needed in her account.

- **Homogeneity and Free Choice**

  We considered the examples (17) – (19) from the paper, which try to show a **parallelism** between homogeneity in plurals and homogeneity in free choice. We observed that this parallelism is not so tenable after all (in (19), the continuation with 'no' seems available, contrary to Goldstein account). We speculated about possible ways to test homogeneity (Tieu et al 2019 might be helpful in this regard) and if an experimental test 'homogeneity vs aloni-account' can be designed.